Granular flows through vertical pipes controlled by an electric field

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The flow of granular nickel particles moving down vertical pipes from a hopper in the presence of a local, horizontal ac electric field is studied experimentally. The flow is initiated by opening the bottom outlet of the pipe after the pipe is fully filled with particles from the hopper. The mass of particles flowing out of the pipe is measured as a function of time by an electronic balance. The time dependence of the steady-state flow rate Q, under each fixed voltage V, is obtained. Depending on the magnitude of V, two types of flow behaviors are observed. For low V ($<V_c=2.0$ kV), a downward-moving interface—separating a dense particle region below it from a low-density region above—exists between the hopper and the electrodes. Two prominent peaks exist in the $Q(t)$ curve for V in the range of 1.4 kV$\leqslant V<V_c$, reulting in two clearly defined flow rates Q_{A_2} and, later in time, Q_B. The particles measured by Q_{A_2} originate from the pipe above the electrodes, and those by Q_B coming initially from the hopper. For high V ($\geqslant V_c$), no interface exists and the whole region between the hopper and the electrodes are densely filled; only one constant flow rate Q_{A_2} is observed. (The precise meaning of Q_{A_2} and Q_B are defined in the text.) The steady-state flow rates Q_{A_2} and Q_B measured for each V, are plotted as a function of V. The flow rate Q_{A_2} is a monotonically decreasing function of V, which can be approximately fitted by a power law, with an exponent of -0.8, while Q_B is found to be voltage independent. These features result from a competition between the blocking effect of the electric-field region and the gravity-driven pushing effect from the hopper outlet. The local electric field is able to retard the downward movement of a dense column existing above it, but is ineffective in doing so when the column above is dilute in density.

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I. INTRODUCTION

Granular matter is a subject of intense interest [1–5] in recent years. In this field, many important topics in nonlinear physics [6]—such as pattern formation [5], solitons [7], chaos [8], and cellular automata [9–11]—were studied. In particular, nonlinear waves in granular flow have been observed and computationally simulated [12–19].

This discrete, compressible system has distinct features when compared with classical fluids. Density fluctuation is an important character of granular flow and has been broadly noticed. Many interesting phenomena related to it were observed in different experiments using x-ray imaging [19], spatiotemporal diagrams [14], and light detector [15]. Intermittent and kinematic shock wave [12] was found in a small-angle two-dimensional funnel when the funnel angle was changed. Different kinds of wave regimes [14] in the vertical pipe were observed when the mass-flow rate was changed by adjusting the stopcock at the bottom end of the tube. The power spectra of density waves were shown to assume a stable power-law form, when the air outflow rate was controlled [15,17]. Jamming phenomenon of granular flow in a two-dimensional hopper was studied experimentally [20].

In this paper, a new granular flow control mechanism—by applying a local, ac electric field—is introduced to study the nickel particle flow in a vertical pipe attached to a hopper. Different density patterns in the flow were observed at different electric-field strengths. Due to the dipole-dipole interaction induced by the electric field, particle clusters were formed near the field and the granular flow rate was reduced nonlinearly with the applied voltage.

II. EXPERIMENTAL SETUP AND METHOD

A hopper with open angle 60° is joined to a short tube of 9 cm long with inner diameter of 7 mm. The tube is then joined to a long pipe of length 85 cm and inner diameter 3 mm. The vertical length between the tube and the pipe is 1 cm (see Fig. 1). All these parts are made of glass. The presence of the short tube practically prevents the flow rate in the pipe from being influenced by how particles are piled up in the hopper.

Most of the particles are spherical in shape (of average diameter 0.25 mm). The nickel particles are metallic; however, they are covered with a thin film of oxide after exposed in air for a long time. The particles are thus electrically insulated from each other.

Two copper electrode plates of vertical length 10 cm, and 1.2 cm in width, are fixed upon the outside wall of the long pipe. The distance between the two electrodes is 5 mm, which is equal to the outer diameter of the long pipe. The upper ends of the electrodes are at a distance of 60 cm from the top of the pipe (see Fig. 1). An ac electric voltage V of 50 Hz in frequency is applied across the electrodes.

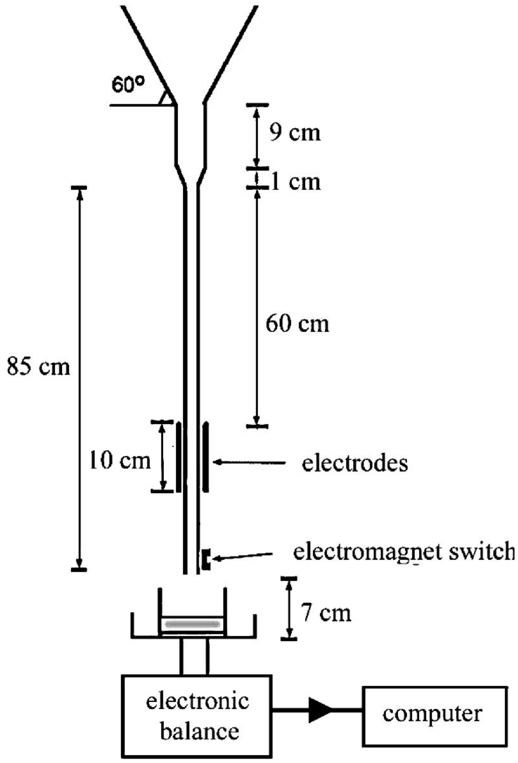

FIG. 1. Sketch of apparatus.

In our experiments, the applied voltage V is fixed in a range of 0–6 kV. The voltage V is increased by 0.2 kV for each different run. For each fixed voltage V applied across the two electrodes, the total mass on the balance M is measured as a function of time t.

Since nickel is a weak ferromagnetic element, we have used a magnet as a stopper to block or initiate the flow. In fact, an electromagnet (placed 1 cm above the bottom end of the pipe, outside of and in contact with the glass pipe) is used. In the beginning of the experiment, we turn on the control current of the electromagnet. We then fill the hopper and the pipe with the nickel particles, up to a fixed height in the hopper. The magnet keeps the particle column from falling out of the pipe. When the control current is switched off, the particles fall down from the pipe and are collected in a container placed on an electronic balance. A thin layer of nickel particles is initially placed inside the container to absorb the impact of the falling masses. The balance sends data to a personal computer at a time interval of 0.4 sec. The resolution of the balance is 10^{-3} g, and the maximum mass allowed by the balance is 200 g.

During the experiments, the humidity is maintained at 47–55 %. This is one of our efforts to reduce the static electric caused by low humidity and, at the same time, to prevent the formation of particle aggregations due to high humidity.

III. EXPERIMENTAL RESULTS

For each experimental run, the mass M collected by the electronic balance as a function of time t is given in Fig. 2. At time $t=t_0$, the measured mass of 40 g is the total mass of

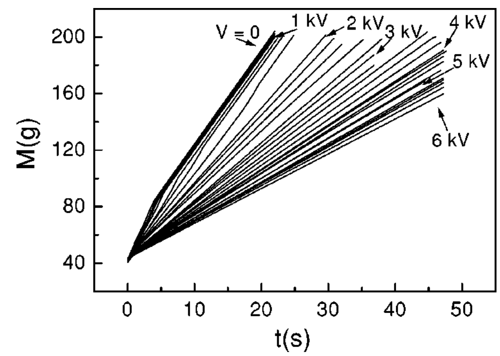

FIG. 2. Dependence of mass M on time t. The voltage difference between two adjacent curves is 0.2 kV.

the container and the initial thin layer of particles placed there. After the flow reaches the balance, the recorded mass M increases with time. At a different voltage the slope of the recorded mass vs time curve is different, as can be easily seen in Fig. 2. The lower the voltage, the faster the flow. It is an immediate evidence that the application of an external electric field is able to affect and retard the granular flow.

The flow rate Q ($\equiv dM/dt$), as a function of t, is obtained from the slope of the $M(t)$ curve, and is plotted in Fig. 3 with V as a parameter. For viewing convenience, each $Q(t)$ curve at different voltage has been shifted vertically upward by 10 g/s from the one below it. When the voltage is zero, the flow rate increases from zero rapidly and fluctuates for a

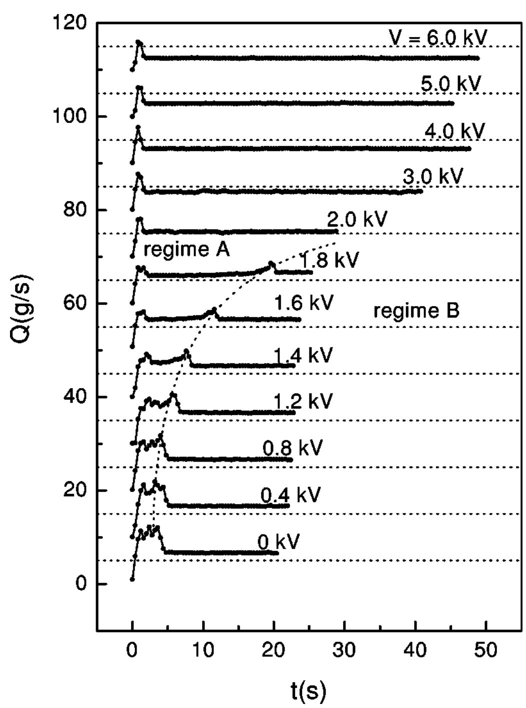

FIG. 3. Steady-state flow rate $Q(t)$, obtained from the slope of $M(t)$ at large t in Fig. 2. For viewing convenience, each of the curves with $V \geqslant 0.4$ kV has been shifted upward by 10 g/s from the curve below it. Each horizontal line represents $Q=5$ g/s, and is drawn to guide the eye to show that Q_B does not change, while Q_{A_2} decreases with voltage. A dotted line connects all the second peaks separating regime A from regime B in the $Q(t)$ curves.

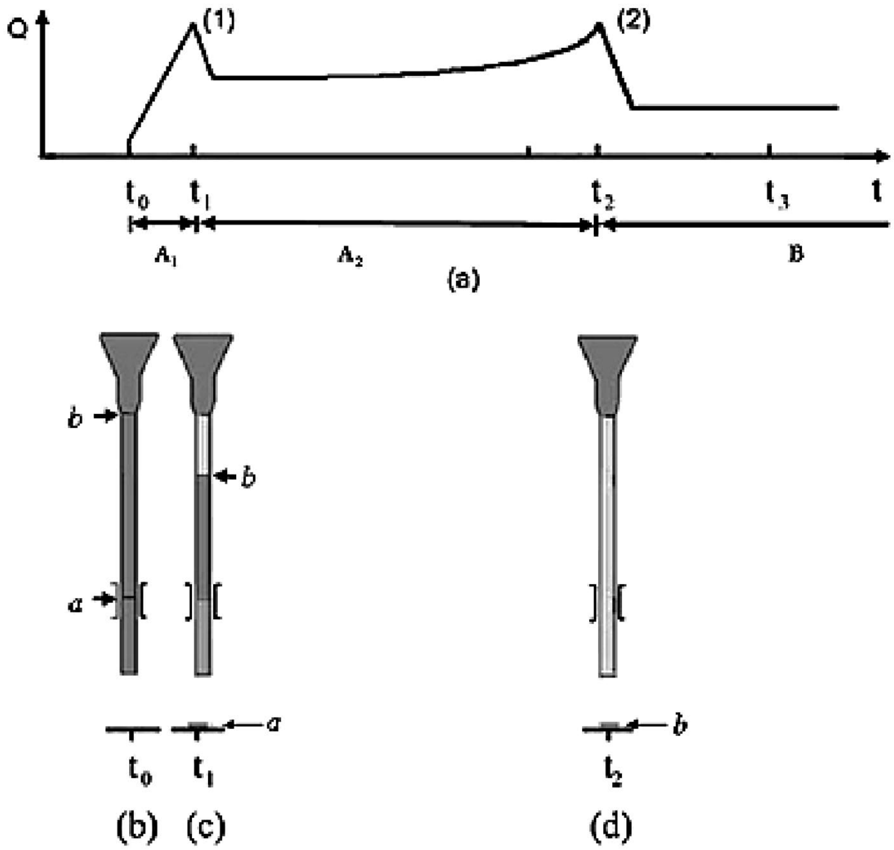

FIG. 4. Sketch of the flow conditions at different time intervals in the $Q(t)$ curve at intermediate voltages $(1.4\ \mathrm{kV} \leqslant V \leqslant 1.8\ \mathrm{kV})$. The dark (light) gray area represents a dense (dilute) region in which the particle density is high (low). (a) Definitions of regime A_1, A_2, and B. (b) At time t_0, when the flow is first initiated. Particles are filled densely in the pipe. (c) At time t_1, particles at layer a reaches the balance; particles in layer b moves downward to somewhere between the top of the pipe and the electrodes, and becomes an interface in effect. (d) At time t_2, layer b reaches the balance, flow in the pipe is dilute in density.

few seconds before it goes steady. As the field is increased, the fluctuating part of the curve expands and eventually separates into two distinct peaks at about $V=1.4$ kV. The first peak stays at the same location, while the second peak moves to the right when V is increased. At $V \geqslant 2.0$ kV, the second peak disappears (see Sec. IV for an explanation). At the same time, the constant flow rate beyond the first peak decreases continuously as V increases, as is shown in Fig. 3.

IV. DISCUSSION

A. Time dependence of the flow rates

As shown in Fig. 3 and sketched in Fig. 4(a), the time-dependent flow rate $Q(t)$ curve can be separated into three regimes A_1, A_2, and B. The time location of the first peak is called t_1; the one for the second peak is called t_2. Regime A_1 is defined as the time interval between $t=0$ and $t=t_1$, regime A_2 as the part between t_1 and t_2, and regime B the part beyond t_2. At $V<1.4$ kV (see curves in Fig. 3), the second peak is too close to the first peak, so that the two peaks can hardly be distinguished. Thus, regime A_2 is buried in the fluctuation of the two partly overlapped peaks and Q_{A_2} is thus not measurable. (Here, Q_{A_2} is defined as the Q at the middle point between t_1 and t_2.) For these curves, regime B is the only regime where a measurable rate—the ‘‘flat’’ (steady) part of the $Q(V)$ curve—may be obtained. On the other hand, for $V>1.8$ kV only one peak remains; the ‘‘flat’’ part corresponds to the flow rate in regime A_2. There is no regime B since the second peak no longer exists. Only in the intermediate V range $(1.4\ \mathrm{kV} \leqslant V \leqslant 1.8\ \mathrm{kV})$, flow rates in both regimes A_2 and B are measurable.

The three regimes described above may be understood as follows. In our experiments, the hopper and the pipe above the stopper are filled with nickel particles before the flow is initiated at time $t=t_0$, as shown in Fig. 4(b) by the dark gray color. Particles in this system may be separated into three regions according to their initial location: (i) particles in the pipe below the electrodes, (ii) particles in the pipe above the electrodes, and (iii) particles in the hopper. These three regimes can be imagined to be separated by two layers of particles, which are called layer a and layer b, respectively [see Fig. 4(b)]. Particles in these three regimes flow at different rates. The location of layer a is within and slightly below the top end of the electrodes; below layer a, the effect of the electrodes is practically zero or can be ignored. Layer b is at the top of the pipe. Particles in region (i) flow solely under gravitational force. Particles in region (ii) are retarded by the electric field, and thus move down slower than particles in region (i). Particles in region (iii) flow into the pipe at a rate determined by the hopper outlet. Due to these rate differences, as layer a reaches the balance at time t_1, layer b moves downward and reaches somewhere above the electrodes. Layer b actually becomes an interface with particle density above it lower than that below it. Simultaneously, an interface appears at the original location of layer a, with particle density from above denser than that at below, opposite to the case at layer b. (Here and below, in reality, the interface is a region of varying density and is of finite width.

But for simplicity of drawing, it is represented by a sharp line.)

The flow rate increases from $Q(t_0)=0$ rapidly until layer *a* reaches the weighing balance at time t_1 as is shown in Fig. 4(c). From time t_0 to t_1, defined as time regime A_1, particles in the flow comes from region (i); the flow is affected neither by the electric-field blocking effect nor by the hopper outlet pushing effect.

In between time t_1 and t_2 (regime A_2), an interface *b* is observed, which moves from top part of the pipe to reach the weighing balance at t_2. The steady-state flow rate Q_{A_2} is always lower than $Q_{A_1}(t_1)$ due to the retardation effect of the electric field. Therefore, there exists a peak at t_1. The particles flowing at the rate Q_{A_2} come from region (ii). It is a ''dense'' flow, in the sense that the flow comes from a dense column of particles from above the electrodes. Note that in this case, this dense column situates right below a dilute column [see Fig. 4(c)].

At time t_2, particles at the interface *b* reach the weighing balance shown in Fig. 4(d). Beyond time t_2, the weighing balance begins to record mass of particles initially from region (iii). The flow then reaches a steady-state rate Q_B. This flow is called a ''dilute'' flow, in the sense that the flow comes from a column of dilute particles in the pipe below the hopper. (Both Q_{A_2} and Q_B actually measure the flow rate out of the bottom of the electrode region, since the original particles in region (i) below the electrodes flow out already in regime A_1.)

The rising part of the second peak of the flow rate Q_{A_2} near time t_2 is due to the finite width of the interface *b*. In this transition region, particle density varies gradually from dense (below the interface *b*) to dilute (above the interface *b*). The flow retardation effect caused by the electric field is reduced as the flow gets more dilute. Thus, as the interface *b* passes through the electrodes, the measured flow rate Q_{A_2} increases.

B. Critical voltage and the second peak

There are two peaks in the $Q(t)$ curve in Fig. 3. The time location of the second peak t_2 increases as voltage *V* increases. The location of the first (second) peak corresponds to the time that layer *a* (layer/interface *b*) reaches the balance.

The downward-moving velocity v_b of interface *b* in the pipe is roughly proportional to the inverse of Δt ($\equiv t_2-t_0$). From the location of t_2 for each *V* in Fig. 3, Δt vs *V* is obtained and plotted in Fig. 5, which is a straight line and vanishes at $V=V_c=2.0\pm0.1$ kV. At the critical voltage V_c, $\Delta t\to\infty$, indicating that the second peak in the $Q(t)$ curve no longer exists, as confirmed by Fig. 3. Physically, for $V \geqslant V_c$, the electric field is strong enough to retard the flow coming from above the electrodes so that the density above layer *b* is the same as that below the layer; a dense column appears above the electrodes. In other words, even though layer *b* indeed moves downward, it no longer becomes an interface as in the case of $V<V_c$.

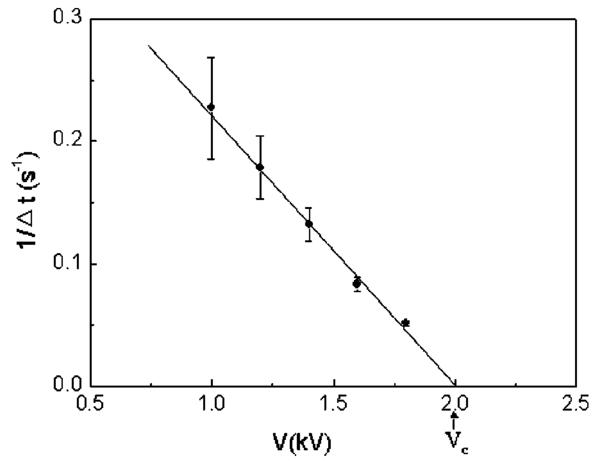

FIG. 5. Inverse of Δt ($\equiv t_2-t_0$) vs voltage *V*, fitted to a straight line vanishing at a critical voltage $V_c=2.0$ kV.

For $V<V_c$ when layer *b* is indeed an interface, velocity v_b depends on the flow rate difference of particles going into and out of the interface. It may be expressed, as sketched in Fig. 6(a), to be

$$v_b=[Q_{A_2}(V)-Q_{B'}]/[(\rho_{A_2}-\rho_B)\Phi], \tag{1}$$

where Q_{A_2} is essentially the flow rate of the particles below the interface *b*; $Q_{B'}$ is the particle flow rate through the hopper outlet; ρ_{A_2} is the flow density below the interface *b*; ρ_B is the flow density above the interface *b*, and Φ is the cross-sectional area of the pipe. The rate Q_{A_2} is controlled by the applied electric voltage *V*. Since V_c is defined to be the voltage at which $(\Delta t)^{-1}=0$, or equivalently, $v_b=0$, it follows from Eq. (1) that V_c is also given by $Q_{A_2}(V_c)=Q_{B'}$, i.e., when layer *b* is no longer an interface, as expected. Under this condition, the second peak in the $Q(t)$ curve disappears. The fact that Fig. 5 gives $V_c=2.0$ kV is in agreement with our experimental observation that at and above $V=2.0$ kV, no down-moving interface *b* is observed in the pipe.

A rough estimate of the critical voltage V_c is given here. At $V\geqslant V_c$, an ''interface'' is formed across the electric-field region in the pipe. This interface is continuously formed and broken in time, so that flow from above can be retarded and pass through at the same time. (At high enough voltage, a dynamically maintained arch is sometimes observed [21].) For simplicity, let us assume that this interface is a horizontal flat surface. In the static case, such a surface is possible when a particle is able to adhere horizontally to another particle. This happens when the interparticle frictional force (proportional to E^2 in the case of dipole-dipole interaction) acting on a particle is equal to or greater than its weight. An order-of-magnitude estimate gives a critical $E_c\cong0.35$ kV/mm, corresponding to $V\cong1.75$ kV in our system. The magnetic field induced by this ac electric field is estimated to be about 10^{-11} Wb/m^2. The estimated energy density ratio of the magnetic field compared to that of the electric field is thus of the order of 10^{-15}. Hence, the effect of the magnetic field induced by the ac electric field can be ignored.

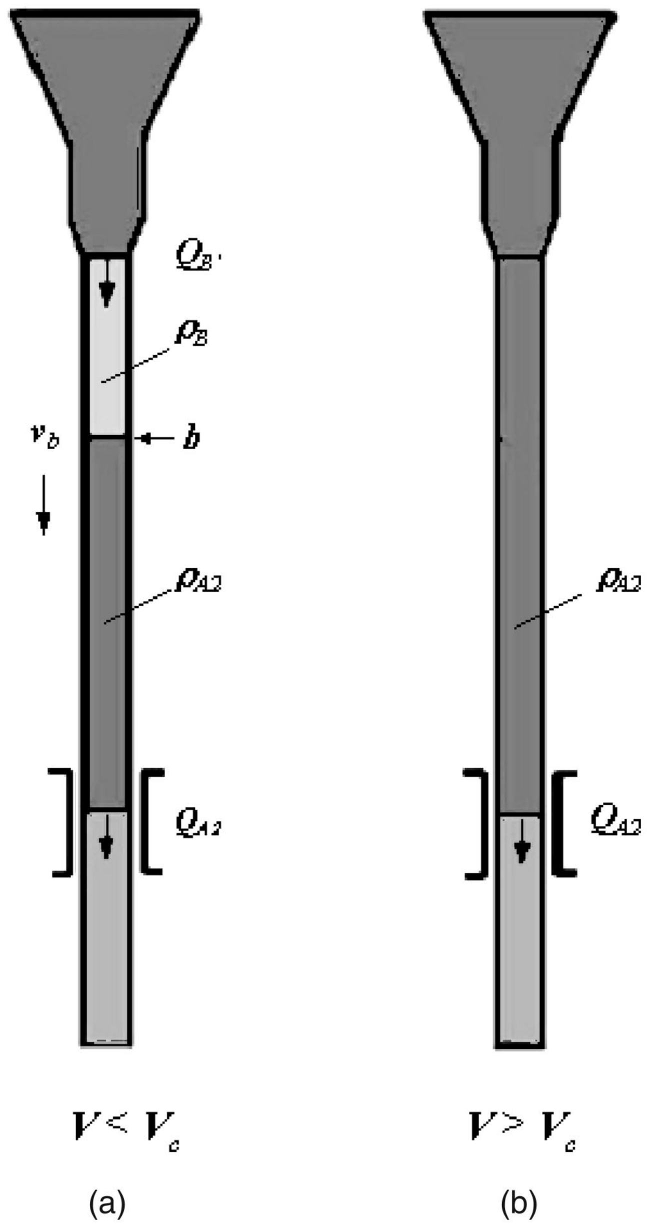

FIG. 6. Sketch of granular flows in regime A_2 (a) when $V < V_c$ and (b) when $V > V_c$. In (a), the interface b exists in the pipe. See text for definitions of the symbols. In (b), the interface b does not exist in the pipe.

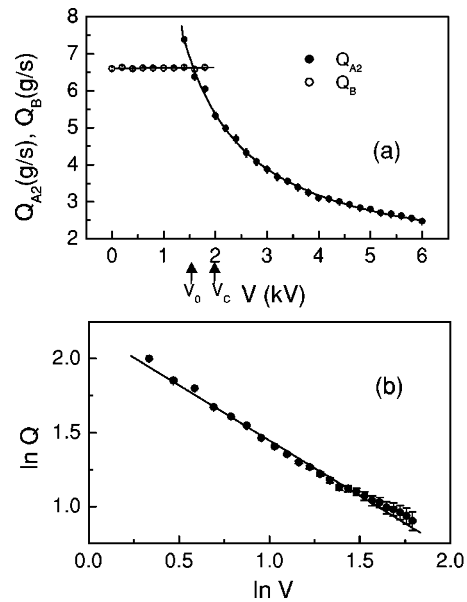

FIG. 7. (a) Flow rates Q_{A_2} and Q_B vs V. Lines connecting data points are drawn to guide the eye. Q_B is essentially constant; Q_{A_2} decreases monotonically with increasing V. Regime B disappears at a voltage $V_c = 2.0$ kV, which is obtained from Fig. 5. The two curves meet at V_0. (b) A log-log plot of Q_{A_2} vs V. The linear fit gives a power law of $Q_{A_2} \sim V^{-0.8}$.

C. Dependence of the flow rates upon electric voltage

The flow rates Q_{A_2} and Q_B measured from Fig. 3, as a function of V, are plotted in Fig. 7(a). As shown in Fig. 4(d), particles initially in region (iii) (particles in the hopper) flow through the pipe and reach the balance at a rate Q_B. The flow rate Q_B appears and is measurable only below the critical voltage 2.0 kV (see Fig. 3). From Fig. 7(a), Q_B is apparently independent of V. The flow rate Q_{A_2}, however, appears and is measurable at all voltages as long as the two peaks in the $Q(t)$ curve are distinguishable. Q_{A_2} is the rate of particles coming from region (ii) (particles in the pipe), as measured at the balance. As shown in Fig. 7(a), Q_{A_2} is voltage dependent; it decreases monotonically with V. While the horizontal Q_B curve terminates abruptly at V_c for a good physical reason, there is no physical reason that the Q_{A_2} curve cannot be extended to the low V region. What is needed is better resolution in the $Q(t)$ curve in Fig. 3.

The two curves Q_{A_2} and Q_B cross each other at a voltage $V_0 \simeq 1.6$ kV. What happens physically near V_0 is very interesting. By definition, Q_{A_2} precedes Q_B in time in the measurement. The fact that $Q_{A_2} > Q_B$ at $V = 1.4$ kV is not surprising, since what it implies is that the flow rate out of the hopper is lower than that out of the pipe, consistent with the picture that there exists an interface b [see Fig. 4(c)]. What is surprising is that at $V = 1.6$ and 1.8 kV, respectively, $Q_{A_2} < Q_B$ is recorded. The lowering of Q_{A_2} at these two voltages implies that the voltage is large enough to retard the dense column of particles in region (ii). The paradox arises from the fact that if Q_B is greater than Q_{A_2}, then how come an interface b (with dilute particles above it) can exist, if Q_B is interpreted as the flow rate out of the hopper outlet? The answer to this puzzle rests on the fact that during the flow when Q_{A_2} is measured, the flow rate out of the hopper is not Q_B, but $Q_{B'}$. $Q_{B'}$ is lower than Q_B because of the air trapped above the interface b; the air will tend to slow down the particles falling out of the hopper. This air effect is more prominent for V less than but close to V_c, because particles flowing out of the hopper see a hardly moving interface b, i.e., $v_b \simeq 0$. A log-log plot of Q_{A_2} vs V is given in Fig. 7(b), giving a power law of $Q_{A_2} \sim V^{-0.8}$.

V. CONCLUSION

The influence of ac electric voltages on the flow of nickel particles from a hopper into a vertical pipe is studied. The applied horizontal electric field is shown to be able to retard the granular flow. The flow rate is controlled at two vertical positions: at the hopper outlet and near the upper end of the electrodes. Three flow rate regimes A_1, A_2, and B are observed and explained. At relatively weak electric fields when $V<V_c$ ($=2.0$ kV), a transition of the granular flow in the pipe from regime A to regime B is observed. The higher V is, the longer regime A_2 lasts. When $V>V_c$, regime B of the flow disappears and the flow stays in regime A_2. The flow rate at regime B, Q_B, is practically unaffected by V. The flow rate of regime A_2, Q_{A_2}, decreases with V monotonically with a power law, which shows that the applied electric field can indeed retard the nickel granular flow. At voltage V_0 slightly below V_c, Q_{A_2} equals to Q_B. Air column effect is believed to be important near V_0.

The retardation effect of the electric field may be attributed to particles forming clusters at the two sidewalls near the electrodes, when V is large enough. Due to the inhomogeneity of the electric field, the polarized particles are pulled towards the two sidewalls; the friction between the particles and the walls is enhanced. The clusters near the wall block the movement of some particles and effectively reduces the cross-sectional area of the tube at the electrode region, resulting in retardation in the flow.

The flow features are explained as coming from a competition between the blocking effect due to the electric field and the pushing effect due to gravity. Note that the whole pipe is saturated with dense particles initially. In regime A_2, the local electric field is able to retard the downward movement of the dense column of particles above it; this flow is called a ‘‘dense’’ flow. Within this regime, depending on the magnitude of V, two kinds of behaviors are observed. (i) For $V<V_c$, the flow rate from the hopper outlet is less than that below the electrode region; a dilute column exists right below the hopper outlet. As time increases, this column extends in length from zero to throughout the whole pipe. The flow becomes a ’’dilute’’ flow—this is regime B. The voltage V (<2.0 kV) is not able to retard this dilute flow in regime B, because the particles are not close enough to each other that the induced dipole interaction is too weak to bind the particles together to form clusters. (ii) For $V>V_c$, the flow rate below the electrodes decreases so much (compared to that from the hopper outlet) that a dilute column below the hopper is never formed, leading to the absence of regime B. The flow in regime A_2, a dense flow, and is retarded by the electric field, as in case (i).

An interesting question remains on whether a high enough voltage exists that is able to retard a dilute flow. Within our experiments reported here, this question cannot be answered. The reason is that because the pipe is initially densely filled due to the placement of the switch at the bottom of the pipe, a dilute flow does not exist for $V>2.0$ kV. Therefore, we cannot test the effect of V on a dilute flow here, for $V>2.0$ kV. In fact, in a separate experiment in which dilute flows are created by effectively placing the switch right under the hopper [22], the dilute flow is retarded by high enough voltages.

ACKNOWLEDGMENTS

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