Global Nature of Dilute-to-Dense Transition of Granular Flows in a 2D Channel

M. Hou,* W. Chen, T. Zhang, and K. Lu
Institute of Physics, Chinese Academy of Sciences, Beijing 100080

C. K. Chan
Institute of Physics, Academia Sinica, NanKang Taipei 115
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The dilute-to-dense transition of granular flow of particle size d_0 is studied experimentally in a two-dimensional channel (width D) with confined exit (width d). Our results show that with fixed d and D there is a maximum inflow rate Q_c above which the flow changes from dilute to dense and the outflow rate $\langle Q(t)\rangle$ drops abruptly from Q_c to a dense rate Q_d. A rescaled critical rate q_c is found to be a function of a scaling variable λ only: $q_c \sim F(\lambda)$, where $\lambda \equiv \frac{d}{d_0}\frac{d}{\mathsf{D}-d}$. This form of λ suggests that the dilute-to-dense transition is a global property of the flow, unlike the jamming transition which depends only on $\frac{d}{d_0}$. Furthermore, the transition is found to occur when the area fraction of particles near the exit exceeds a critical value which is close to 0.65 ± 0.03.

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The phenomena of crowding or jamming are common experiences in our daily lives. However, advances in the understanding of these processes have only begun recently after the physics of granular materials have been under intense investigations [1,2]. It is now believed that the diverse phenomena of traffic flow, pedestrian flow, and floating ice [3–5] are related to the nonlinear behaviors of granular materials which can exhibit both solidlike and fluidlike behaviors [6–8]. These peculiar properties [8,9] give rise to at least three important "states" in granular flows, namely, the dilute flow, dense flow, and the jammed state. The phenomenon of crowding can then be understood as a transition from dilute to dense flows and that of jamming is a transition from dense flows to a jammed phase.

Obviously, the nature and properties of these transitions are governed by the interactions among the granular particles in the flow. In principle, these transitions and states can be well characterized when these interactions are known such as in the cases of equilibrium systems [10,11]. Unfortunately, since interactions among the granular particles are highly nonlinear and characterized by dissipations which can be density and even history dependent, it is not surprising to find that these transitions and their states are still not well understood. Recently, there has been some progress in the understanding of the transition from dense to jammed states [12,13] and even an equilibriumlike jamming phase diagram has been proposed [14–16]. However, still very little is known about the dilute to density flow transition.

An important characteristic of the dense-to-jam transition is that there is usually only one dominant length scale in the problem, namely, the size of the grains (d_0) [12]. In contrast, it seems that the global scale of the order of the size of the system is important for the dilute-to-dense transitions. For example, a small bottleneck of the size of the system can induce the dilute-to-dense transition [17]. In this aspect, the dilute-to-dense transition is similar to transitions in hydrodynamic flows. In the case of hydrodynamic flows, it is well known that different Reynolds numbers are associated with different flow configurations to characterize the flow as laminar or turbulent. The Reynolds number is a global parameter which scales with the system sizes. Intuitively, in the case of dilute-to-dense transitions, a similar global scaling parameter might exist. If such a scaling parameter can be found, its scaling form will probably provide a better understanding of the nature of the dilute-to-dense transitions.

In this Letter, we report our results on experiments carried out in a 2D channel to look for a relation between the dilute-to-dense transition and system parameters. We find that the critical flow rate Q_c at the transition can be well characterized by the scaling variable $\lambda \equiv \frac{d}{d_0}\frac{d}{\mathsf{D}-d}$ where the system parameters are the channel width (D), the opening of the channel (d) and the diameter of the grains (d_0). Q_c is the maximum dilute flow rate above which the flow changes from dilute to dense. The scaled critical flow rate $q_c(\equiv Q_c/(\mathsf{D}/d_0))$ is found to be a function of a scaling variable λ only as $q_c \sim F(\lambda)$. This form of λ suggests that the dilute-to-dense transition is a global property of the flow, unlike the jamming transition which depends only on $\frac{d}{d_0}$. Furthermore, the transition is found to occur when the area fraction of particles near the exit exceeds a critical value which is close to 0.65 ± 0.03.

Our experiments are performed in a two-dimensional (2D) channel with an inclination angle of 20°. The 2D channel is established on a metal plate between two glass plates separated by specially shaped metal spacers to form a test section and a hopper as shown in Fig. 1. The gap between the two glass surfaces is kept at 1.2 mm (2.2 mm) to ensure an almost single-layer flow of steel beads of diameter $d_0 = 1 \pm 0.01$ mm (2 ± 0.01 mm) which are stored in the hopper at the top of the channel.

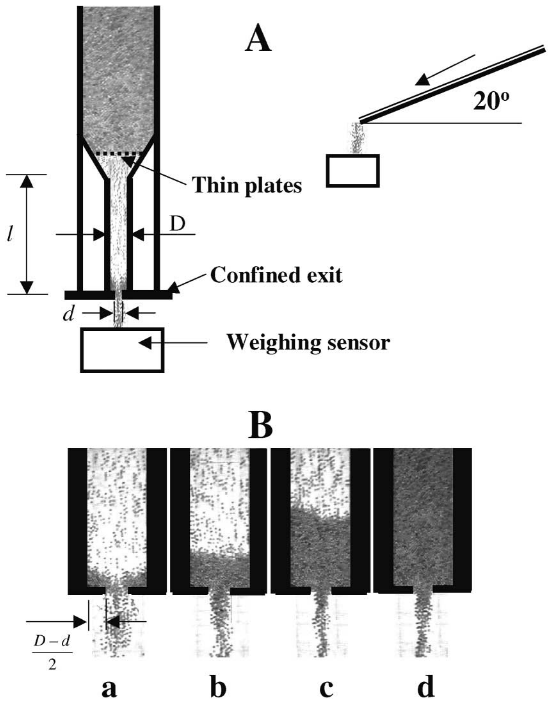

FIG. 1. (A) Top and side views of the inclined channel. (B) Photos of dilute-to-dense flow transition in a time sequence

The hopper, with an open angle of 60°, is connected to a test section of width D = 24 mm and length $l = 500$ mm. At the end of the test section, there is an exit of width d which is controlled by micrometers to a precision of 0.01 mm. Granular flows in the test section are initiated by allowing the steel beads in the hopper to fall by gravity. A thin plate with a number of uniformly distributed holes is inserted close to the exit of the hopper to control the inflow rate and ensure the uniformity of particle distribution across the test section. The total mass M of the beads falling out of the exit is measured as a function of time t by an electronic balance with sensitivity of 0.02 g and a weighing period of 0.02 s. The flow rate $Q(t)$ is obtained by the slope of the recorded $M(t) curve$.

The flow in the test section initiated from the hopper is dilute and accelerating. The typical velocity of the steel beads close to the exit is 1.0 ± 0.1 m/s. Since d is smaller than D, two wedges (heaps) will be formed at both sides of the exit with a base length $(\mathrm{D} - d)/2$. If the inflow rate Q_0 is small or d is not too small, there will be no net accumulation of beads in the test section other than the two wedges and $\langle Q(t)\rangle = Q_0$. This is the regime of dilute flows as shown in (a) of Fig. 1(B). However, for a given Q_0, there will be a critical d_c below which there will be net accumulation of beads in the test section. In other words, if d is decreased systematically, there will be a sudden drop of $Q(t)$ when d_c is reached. This accumulation of beads will proceed until the whole test section is filled with beads and this is the regime of dense flows. The process of this dilute-to-dense transition induced by reducing d is shown in Fig. 1(B), which is recorded by a video camera.

Figure 2 shows the d dependence of $\langle Q(t)\rangle$ when d is reduced or increased systematically for a given Q_0. When d is large (point A), $\langle Q(t)\rangle = Q_0$, the flow is dilute. It can be seen that $\langle Q(t)\rangle$ remains practically independent of d when d is larger than a critical size d_c (A to B in Fig. 2). When d_c is reached, the dilute flow turns to dense flow and the flow rate can be reduced instantaneously by several times to drop from Q_0 to Q_d (B to C). After the transition, Q_d decreases monotonically with reducing d (C to D). The flow jams when d is about the size of four-particle diameters, where permanent arching occurs to cause jamming of the flow [12] (D to E). When increasing d from the jammed phase, the flow starts as dense flow, and the rate $\langle Q(t)\rangle$ increases gradually with increasing d until Q_d reaches Q_0 and turns back to dilute flow as shown by triangle points in Fig. 2 (D through C to A). There is no sudden increase of the flow rate at the transition from dense to dilute flow. We have checked that the dense flow rate curve $Q_d(d)$ (the CD part of curve ACD) follows the Beverloo empirical equation $(d - kd_0)^{3/2}$ with $k = 4$ [18,19] and is independent of Q_0.

If the experiment of Fig. 2 is repeated with different Q_0, a family of paths similar to $ABCD$ of Fig. 2 will be formed, shown as dotted lines in Fig. 2. An important characteristic of this family of lines is that d_c decreases with Q_0 shown as broken line BF. That is, for a given d, there will be a Q_0 [denoted as $Q_c(d)$] at which a dilute-to-dense transition will occur. Figure 2 is the result of experiments with fixed D but obviously $Q_c(d)$ will also depend on D and d_0. The $Q_c(d)$ curves for various D and d_0 have been measured and shown in Fig. 3. Four D's, D = 30, 25, 20, and 15 mm, are tested for $d_0 = 1$ mm

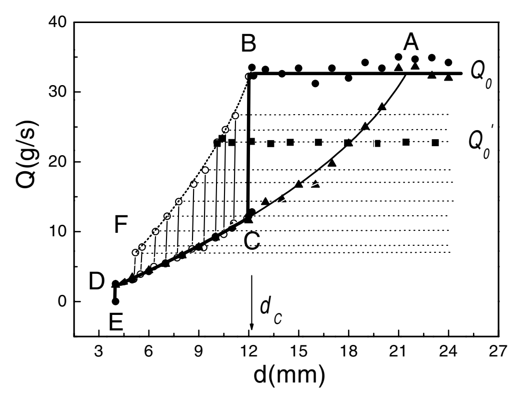

FIG. 2. At a given inflow rate Q_0, the outflow behaves differently when the flow is dilute or dense. For a dilute flow, a transition from dilute to dense occurs at a critical opening size d_c, and the outflow follows curve $ABCDE$. For a dense flow, as d increases the flow follows curve DC and is extended to A with no abrupt change. For different Q_0, the transition occurs at different d, as shown by dashed lines. The curve BF determines the optimal outflow rate at any given Q_0. For clarity, only two dilute-flow experimental data sets are shown as solid circles and squares.

particles, and two D's, $\mathsf{D} = 40$ and 30 mm, are tested for 2 mm particles. In Fig. 3 the upper six curves are $Q_c(d)$'s and the lower six curves are the corresponding $Q_d(d)$ curves for particles $d_0 = 1$ and 2 mm. If the dilute-to-dense transition scales with the system size, one might expect that the exit width d should be scaled by channel width D as $Q_c(d)/\mathsf{D} = F(d/\mathsf{D})$ for some scaling function F. The inset of Fig. 4 shows such a plot and obviously such scaling does not hold. Instead, a new scaling variable $\lambda \equiv (\frac{d}{\mathsf{D}-d})(\frac{d}{d_0})$ is found to collapse the scaled critical flow rate $q_c(\equiv Q_c(d)/(\mathsf{D}/d_0))$ into a single scaling curve as shown in Fig. 4. The result in Fig. 4 can be expressed as $q_c = q_0 F(\lambda)$, where q_0 is some constant. It means that the maximum flow rate of a dilute flow of a system of d and D is not the same as that of a system of $2d$ and $2D$ although their aspect ratio is the same.

A remarkable feature of the dilute-to-dense transition is that there are strong fluctuations in $Q(t)$ when d is set to close to d_c. Direct observations of the motions of the beads close to the exit reveal that there are avalanchelike events taking place at the two wedges on either side of the exit. After the wedges have been built up by the incoming flux, flows of surface layer in the form of avalanches will occur. The two wedges effectively act as collectors of incoming flux and direct them to the exit through the surface flow. Therefore, there are strong density fluctuations (in terms of area fraction p) near the exit. When the discharge from the two wedges meet at the exit, the outflow can sometimes be blocked, leading to dense flow. This blockage is intermittent if the incoming flux is not large enough and therefore producing strong fluctuations in $Q(t)$. However, if the incoming flux exceeds Q_c, the blockage becomes permanent and there is accumulation of the beads in the test section until the whole section is filled. Particle density ρ close to the exit can be determined from video pictures. It is found that a dilute-to-dense transition will occur if ρ exceeds a critical value ρ_c. In our experiments, it seems that the value of ρ_c always corresponds to that of an area fraction of 0.65 ± 0.03 and is independent of Q_c, d, or D.

Following the observations discussed above, the flow across the test section can be divided into three regions: the central part with length d and the two wedges with base length $\frac{\mathsf{D}-d}{2}$. If v_0 and ρ_0 are the velocity and density of the dilute flow initiated by the hopper just before particles reaching the wedges, the incoming flux on the wedge is $v_0\rho_0\frac{\mathsf{D}-d}{2}$. The outgoing flux of the wedge will be carried away by the fluidized surface layer mentioned as $v_e\rho_c\delta$, where δ and v_e are the depth and characteristic velocity of the surface layer, respectively. Conservation of flux gives $\delta = \frac{v_0\rho_0}{v_e\rho_c}\frac{\mathsf{D}-d}{2}$. In this model, δ will increase with the incoming flux or ρ_0. When these two top layers meet at the exit, we have $\delta\beta = d/2$, where β is some geometric factor which takes care of the angle of repose. Therefore, at the transition, we have $f_c \equiv (v_0\rho_0)_c = v_e\rho_c\beta^{-1}\frac{d}{\mathsf{D}-d}$ or $\frac{f_c}{f^*} = \frac{v_e}{v^*}\frac{d}{\mathsf{D}-d}$, where $f^* \equiv v^*\rho_c\beta^{-1}$. One can consider v^* as some intrinsic velocity of the problem which is determined by physical properties of the system such as the angle of inclination or coefficient of sliding friction, etc. Presumably $\frac{v_e}{v^*}$ is a function of system parameters D and d. However, for the case of $\mathsf{D} \gg d$, it is reasonable to assume that $\frac{v_e}{v^*}$ depends only on $\frac{d}{d_0}$. In such a case, one expects to see $\frac{v_e}{v^*} \sim \frac{d}{d_0}$ because this is the first order expansion of v_e in terms of d when $d = 0$ gives $v_e = 0$. This later form of $\frac{v_e}{v^*}$ gives $\frac{f_c}{f^*} \sim \lambda \equiv (\frac{d}{d_0})\frac{d}{\mathsf{D}-d}$ which agrees with our result in Fig. 4 for small d or λ. Physically $(v_e/v^*) \sim (d/d_o)$ means that the discharge velocity of the fluidized layer of the two wedges increases with d. Note that v_e for the dense flow, $v_e \sim d^\alpha$ with $\alpha = 1/2$ [19].

It can be seen from Fig. 4 that q_c increases with λ monotonically. Obviously, for a fixed D, Q_c must tend to a

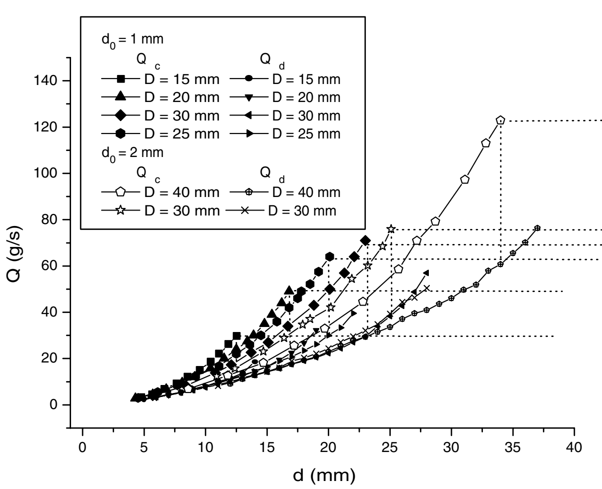

FIG. 3. The Q_c and Q_d vs d of particle size $d_0 = 1$ mm at channel widths $\mathsf{D} = 30$, 25, 20, and 15 mm, and of particle size $d_0 = 2$ mm at channel widths $\mathsf{D} = 40$ and 30 mm.

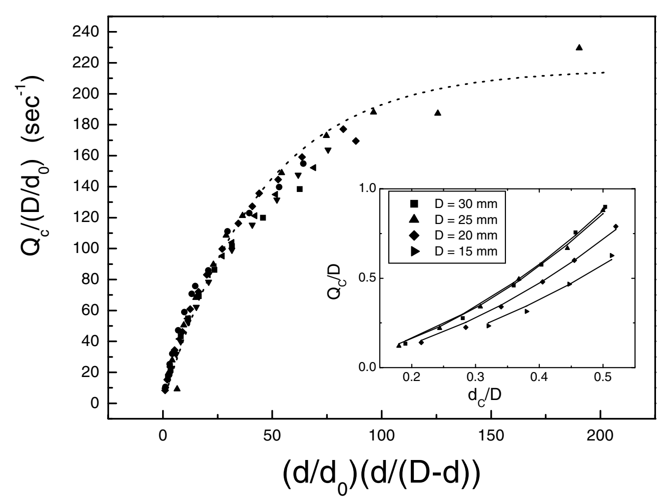

FIG. 4. The rescaled $q_c(= Q_c/(\mathsf{D}/d_0))$ vs a new scaling variable $\lambda(= (d/d_0)[d/(\mathsf{D}-d)])$, while Q_c/D vs d/D are shown in the inset ($d_0 = 1$ mm in this case). The dotted line is a fit of q_c by $q_m(1 - e^{-(\lambda/\lambda_0)})$, where $q_m = 216$ and $\lambda_0 = 45$.

limit for large enough d/d_0 in the experiments because Q is given by $v_0\rho_0\mathsf{D}$ and there must be an upper limit in ρ_0 to still have dilute flows in the test section. If q_m is the maximum dilute flow flux of the channel, we have $q_m = q_0F(\infty)$. One can choose $F(0) = 0$ and $F(\infty) = 1$ to give $q_c = q_mF(\lambda)$. Note that there are two ways for λ to go to ∞. When $d = \mathsf{D}$, the test section is just a straight pipe (2D), q_m is obviously just the maximum dilute flow capacity of the pipe. For fixed d/D, λ goes to ∞ when either d goes to ∞ or d_0 goes to zero. In both cases, we have a continuum limit in which the size of the particles can be neglected. Therefore, our model predicts that q_m is independent of d/D when d/d_0 is large enough. The functional form of $F(\lambda) = 1 - e^{-(\lambda/\lambda_0)}$, which gives the correct form of q_c for small λ, has been fitted to the data in Fig. 4 shown as a dotted line. The scattering of the measured values of Q_c increases as λ increases, which makes it difficult to determine experimentally the limit value of q_c. Presumably, our model discussed above is valid only for small d.

The phenomenological model described above is based on the observation that the transition occurs when the flow density near the exit reaches $p = 0.65 \pm 0.03$. The abrupt change of the flow rate at the dilute-dense transition may be understood as the inelastic collisions of the particles with the two piles near exit. In some aspects, our system is similar to the wedge setup of Rericha *et al.* [20], where shocks identical to those in a supersonic gas are observed when a steady flow passes the wedge. Our system, however, with the piled heaps at the two sidewalls near the exit is equivalent to a system where particles pass two inward "soft wedges," a system more commonly seen in industrial transport of granules. The abrupt change in density, granular temperature, and velocity before and after the dense area near the exit may account for the flow rate drops at the dilute-dense transition, as the flow rate is a function of the product of flow density and the particle velocity. A two-dimensional molecular dynamics simulation using force model similar to [21] is performed to provide the microscopic view of the transition process. Simulation results show that the dense flow and the dilute flow are two different states. When the flow density reaches $p = 0.65 \pm 0.03$, the flow becomes dense flow and the multibody collisions dominate.

In conclusion, we have experimentally obtained a dilute-to-dense transition curve $Q_c(Q_0, d_c)$, at which the flow rate Q_0 drops to Q_d at d_c. Instead of the intuitive scaling relationship d/D, experimental results show that a global scaling variable $\lambda \equiv (d/(\mathsf{D} - d))(d/d_0)$ determines if the flow is dilute or dense at a given Q_0. The scaled q_c can be fitted in an empirical form $q_m(1 - e^{-(\lambda/\lambda_0)})$. The ratio λ/λ_0 determines the critical flux of the system. Intuitively, λ_0 is determined by physical properties of the system, such as the elasticity of the particles, particle size, and the inclination angle of the plate, etc. While the physics of jamming transition is determined by local scales close to d_0, our result suggests that the physics of dilute-to-dense transition is similar to hydrodynamic instability which is controlled by global scales such as $d/(\mathsf{D} - d)$. The discovery of this global scaling property may provide us with ideas of better designs for the transport of granules in industrial processing, and better understanding of similar flow systems such as systems in traffic flows.

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*To whom correspondence should be addressed.
Electronic address: mayhou@aphy.iphy.ac.cn

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