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FACTORS OF THE TRANSITION FROM DILUTE FLOW TO DENSE FLOW IN TWO-DIMENSIONAL CHANNEL

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Granular flow of steel beads from dilute flow to dense flow on an inclined two- dimensional channel is studied. The initial inflow Q_0 is always a dilute flow. A transition from dilute to dense is observed when $D \leq D_c$. In the dense flow the outflow rate (Q) depends only on opening (D), given by $Q = aD^{\frac{3}{2}}$. For different inflow rate Q_0 in the dilute flow case, the relation between the transition critical exit width (D_c) and the channel width (W) is given. And in larger inflow, the critical exit width is approximately in proportion to the square root of inflow rate, $D_c \propto Q_0^{\frac{1}{2}}$.

1. Introduction

Recently, there has been an increasing interest to physicists in granular matter.[1] One of the basic reasons is the granular matter existing extensively in nature and in industrial production, such as granular matter reserving, transportation both which very concern by engineering.[2] On the other hand, the granular matter is a nonlinear and discrete, complex system with particular properties,[3] which is less known. The properties of the granular matter are much a concern of the engineering, the transport of the granular, traffic flow, pedestrian flow, floating ice and etc. A lot of research about the granular matter has recently been reported. For example, Beverloo and his colleagues studied the relation between the mass-flow-rate Q of granular in funnel and the exit width D of the funnel,[6] in the granular flow as dense flow. In the two-dimensional flow, the flow rate can be described as $Q = c\rho\sqrt{g}D^{\frac{3}{2}}$, where C is constant, ρ is the density of the granular, g is the acceleration of gravity. Although the result has been approbated at large, the granular matter flow is still not clear in theory. K. To and his colleagues[5] recently studied the jamming of granular flow in the two-dimensional hopper, their results show that the jamming probability equals approximately to one when the exit width is less than four times of the diameter of the granular. Meiying Hou and her colleagues have observed the

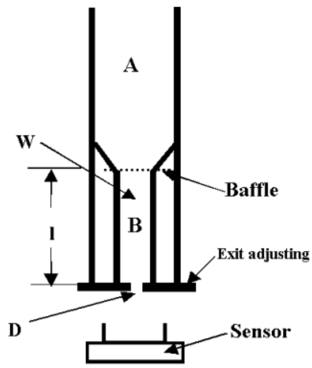


Fig. 1. Schematic diagram of the experimental setup.

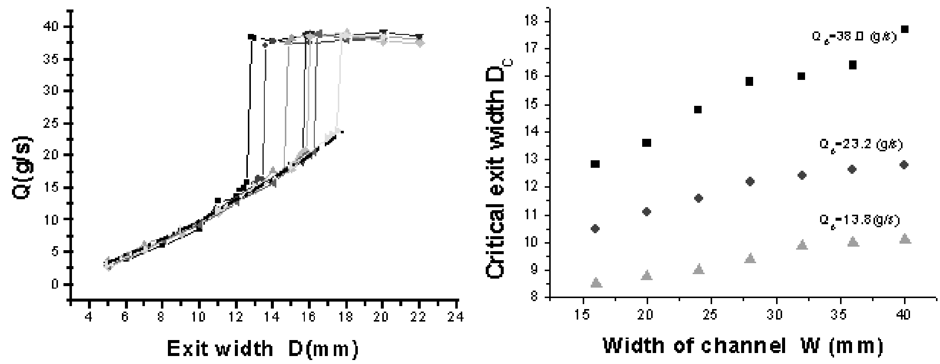
transitions from dilute flow to dense flow by applying a local horizontal ac electric field to granular flows in a vertical pipe recently.[4] Also, in the two-dimensional channel, the relation between the transition of the dilute and dense flow and the exit width have been studied.[7] The results show there is a transition from the dilute flow to the dense flow when the exit width D decreases to a critical value D_c or the inflow rate Q_0 goes up to a critical value Q_c. In this paper, we will discuss the transition from the dilute flow to the dense flow in the two-dimensional channel induced by the channel width, and the relation between the critical exit width D_c and the width of channel W is reported.

2. Experimental Setup

The experiment on this work is completed in the laboratory of the soft and granular matter at Institute of Physics, Chinese Academy of Sciences. The experimental setup is shown in Fig. 1, the two-dimensional channel makes up of two glass plates separated by 1.2 mm, the channel plane with inclined angle of 25°, and the length of the bevel 2.0 m and width 0.6 m. The granular matter, steel beads of diameter $d_0 = 1 \pm 0.01$ mm, are stored in the two-dimensional hopper with an open angle of 60° that is formed by two steel plates, and is connected to the channel, see part A in Fig. 1. The inflow rate Q_0 is controlled by pulling out a thin plate placed as a stopper at the hopper outlet. The width of the channel can be adjusted continually (see part B in Fig. 1). The exit width of the channel D is controlled by micrometers to the precision of 0.01 mm. A weighting sensor with sensitivity of 0.02 g and recording rate of 0.02 s is placed underside of the exit, The mass of beads $M(t)$ falling out of the exit is a function of time that can be recorded and the experimental data will be transmitted to computer instantaneously. The outflow rate $Q = dM/dt$ was obtained by the slope of the recorded $M(t)$ curve.

3. Results and Discussion

At a fixed inflow rate Q_0 (Q_0 = 38.0g/s) for different widths of channel W of the dilute flow, it is recorded the relations between the outflow rate Q and the



(a) (b)

Fig. 2. (a) At a fixed inflow rate Q_0, the relation between the outflow rate Q and the exit width D in different width of channel. From left to right, the width of channel W are corresponding to 16, 20, 24, 28, 32, 36 and 40 mm. (b) there is a different critical exit width D_c for the different inflow rate Q_0 in the same width of channel W

exit-size D (in Fig. 2(a)) are recorded. The seven curves in Fig. 2(a) are the results of different width W of the channel. The widths W equals to : 6.0 mm, 20 mm, 24 mm, 28 mm, 32 mm and 36 mm, from left to right respectively. The results show there is a transition from the dilute flow to dense flow for different widths of channel when the exit width is reduced gradually. The transition is due to the enhanced interaction among the beads while the width D is reduced gradually. When the exit width is large than D_c, the outflow rate Q equals to inflow rate Q_0. However, the outflow rate Q is nonlinear with the exit width d when the exit width D is smaller than D_c. The flow rate Q obeys the follow relation $Q = aD^{\frac{3}{2}}$ when the exit width is smaller than D_c, (see the dashed line in Fig. 2(a)), where $a = 0.315 \pm 0.005$.This is in agreement with the result of Beverloo and his colleagues. Hence, the outflow rate Q is only relation with the exit width in the dense flow. The inflow rate Q_0 and the width of channel W do not seem to have effect on it when the width of channel W is greater than the diameter of the granular. At a fixed inflow rate Q_0, the critical exit width D_c corresponding to the transition from a dilute flow to the dense flow is connected with the width of channel W. The critical exit width D_c is greater when the width of the channel W is larger. The relation between the D_c and W is also a function of inflow Q_0, as shown in Fig. 2(b). The inflow rates Q_0 of the three curves are 38.0, 23.2 and 13.6 g/s, respectively. Figures 2(a) and 2(b) show that the critical exit width D_c is a function of the width of channel W and the inflow rate Q_0. The relation between the Q_0, D_c and the W can be formulated as[7]

$$Q_0 = \rho_c D_c \left[e + (1-e)\frac{D_c}{W} \right] \sqrt{2gl \sin\theta} \,. \tag{1}$$

Here e is the coefficient of restitution that is introduced due to the partly particle collisions near the exit. ρ_c is the critical density of granular for dense flow, g the gravitational acceleration, θ the channel plane with inclined angle, l the length of channel. These parameters are constant. Here, we rewrite the equation as follows

$$D_c = \left[\frac{e^2}{4(1-e)^2} + \frac{Q_0}{(1-e)W\rho_c\sqrt{2gl\sin\theta}}\right]^{\frac{1}{2}} W - \frac{e}{2(1-e)}W\,. \tag{2}$$

From Eq. (2), one can find the relation among the critical width D_c, when the transition from the dilute flow to the dense flow occurs, the inflow rate $Q_0$and the width of the channel W (the W is always larger than D_c). At a fixed inflow rate Q_0, the critical width D_c is linear increasing with the width of the channel W approximately. But at a fixed width of the channel W, the critical exit width D_c is approximately in proportion to the square root of inflow rate Q_0. This is in accordance with the experimental result in Fig. 2(b).

In above results, there is a transition from dilute to dense flow when the exit width D is in decrease. The critical exit width D_c is a connection not only the inflow rate Q_0 but also the width of the channel W. So, in practice, to avoid the jamming, such as traffic flow, floating ice, pedestrian flow and so on, we can control the inflow rate Q_0 or adjust the width of the channel W.

To understand how the critical exit width D_c is influenced by the width of the channel W, we assume that the distribution of particles through the channel is uniform and have a same velocity (including transverse velocity around the base line). The three dashed curves indicate the three region in which the particles going through the critical exit D_c in order of time. In Fig. 3, it shows fewer particles passing through the exit in Δt_1 time interval and more particles going through the exit in Δt_2 interval, and more particles pass the exit in Δt_3interval. This shows

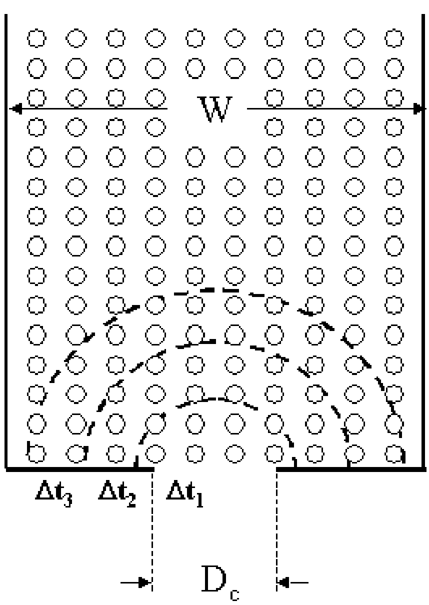


Fig. 3. The distributing of the particles in the channel.

that the flow rate increases gradually at beginning of granular falling, because the portion of beads from farther away reaching the exit is delayed as they going through the channel. Hence, the transition from the dilute flow to the dense flow will occur because the inflow rate Q_0 approaches the critical value in a space of time interval (for example) owing to the accumulation of granular. It causes mostly by the transverse width of channel W. It is found that the critical exit width D_c increases linear with the transverse width of channel W increasing.

4. Conclusion

In this paper we have discussed the experiment on the transition phenomena of granular flow from the dilute flow to the dense flow in two-dimensional channel. The outflow rate Q depends on the inflow rate Q_0 and the exit width D. In the dense flow the outflow rate (Q) depends only on exit width (D) given by $Q = aD^{\frac{3}{2}}$. For different inflow rates Q_0 in the dilute flow, the critical exit width related with the channel width (W) and the Q_0, is discussed. When the inflow rate Q_0 is fixed, the critical exit width D_c will increase linearly with the width of channel W. In greater inflow, the critical exit width is approximately in proportion to the square root of the inflow rate,$D_c \propto Q_0^{\frac{1}{2}}$. These expressions have been validated by the results of the experiments. We have also discussed the transition form the dilute flow to the dense flow influenced by the width of channel W. The results may be helpful in dealing with the practicalities such as granular flow, traffic flow and floating ice etc, with the negative effect caused by the bottleneck will be avoided or abated better.

Acknowledgments

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