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SCALING PROPERTY OF THE DILUTE-DENSE TRANSITION IN 2D GRANULAR FLOWS

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In this paper we report our experimental study of dilute-dense transition in a 2-dimensional granular flow of particle size d_0 and channel width D with confined exit of width d. It is found that a maximum inflow rate Q_c exists, above which the outflow changes from dilute to dense and the outflow rate $Q(t)$ drops abruptly from Q_c to a dense flow rate Q_d. The re-scaled critical rate $q_c(\equiv Q_c/(D/d0))$ is found to be a function of a scaling variable λ only, i.e. $q_c \sim F(\lambda)$, and $\lambda \equiv \frac{d}{d_0}\frac{d}{D-d}$. The form of this new variable λ suggests that the dilute-to-dense transition is a global property of the flow; unlike the jamming transition, which depends only on $\frac{d}{d_0}$. It is also found that this transition occurs when the area fraction of particles near the exit reaches a critical value 0.65 ± 0.03.

Keywords: Granular flow; scaling law; phase transition; hopper flow.

The diverse phenomena of traffic flow, pedestrian flow and floating ice[1−3] are believed to be related to the nonlinear behaviors of granular materials which can exhibit both solid-like and fluid-like behaviors.[4,5] These peculiar properties[6,7] give rise at least to three important "states" in granular flows; namely, the dilute flow, dense flow and the jammed state. The phenomenon of crowding can then be understood as a transition from dilute to dense flows and that of jamming is a transition from dense flows to a jammed phase.

Obviously, the nature and properties of these transitions are governed by the interactions among the granular particles in the flow. In principle, these transitions and "states" can be well characterized when these interactions are known such as in the cases of equilibrium systems.[8,9] Unfortunately, since interactions among the granular particles are highly nonlinear and characterized by dissipations which can be density and even history dependent, it is not surprising to find that these transitions and their states are still not well understood.

An important characteristic of the dense-to-jammed transition is that there is usually only one dominant length scale in the problem; namely the size of the grains (d_0).[10] In contrast, it seems that the global scale of the order of the system dimensions is important for the dilute-to-dense transitions. For example, a small bottle-neck of the size of the system can induce the dilute-to-dense transition.[11] In this aspect, the dilute-to-dense transition is similar to transitions in hydrodynamic flows. In the case of hydrodynamic flows, it is well known that different Reynolds numbers are associated with different flow configurations to characterize the flow as laminar or turbulent. The Reynolds number is a global parameter which scales with the system sizes. Intuitively, in the case of dilute-to-dense transitions, a similar global scaling parameter might exist. If such a scaling parameter can be found, its scaling form will probably provide a better understanding of the nature of the dilute-to-dense transitions.

Here we report our results on experiments carried out in a $2D$ channel to look for a relation between the dilute-to-dense transition and system parameters. We find that the critical flow rate Q_c at the transition can be well characterized by the scaling variable $\lambda \equiv \frac{d}{d_0}\frac{d}{D-d}$ where the system parameters are the channel width (D), the opening of the channel (d) and the diameter of the grains (d_0). Q_c is the critical dilute flow rate above which the outflow changes from dilute to dense. The re-scaled critical flow rate $q_c(\equiv Q_c/(D/d_0))$ is a function of λ only. This suggests that the dilute-to-dense transition is a global property of the flow; unlike the jamming transition, which depends only on $\frac{d}{d_0}$. The transition is found to occur when the area fraction of particles near the exit reaches a critical value $\sim 0.65 \pm 03$.

The setup of this experiment has been described in detail in a previous paper.[12] The experiment is performed in a two-dimensional (2D) channel of length l = 500 mm, and width in a range of 15 mm to 40 mm. The channel is inclined in an angle of 20°. The width d of the exit at the end of the channel is controlled to a precision of 0.01 mm. The steel beads are first stored in a hopper. Granular flows are initiated by allowing the steel beads in the hopper to fall by gravity along the channel. The total mass M of the beads falling out of the exit of the channel is measured as a function of time t by an electronic balance with sensitivity of 0.02 g and a weighing period of 0.02 s. The flow rate $Q(t)$ can then be obtained by calculating the slope of the recorded $M(t)$ curve.

The flow initiated from the hopper is dilute and accelerating. The typical velocity of the steel beads close to the exit is 1.0 ± 0.1 m/sec. Since d is smaller than D, two wedges (heaps) will be formed at both sides of the exit with a base length $(D - d)/2$. If the inflow rate Q_0 is small or d is not too small, there will be no net accumulation of beads in the channel other than the two wedges and $\langle Q(t) \rangle = Q_0$. This is the regime of dilute flows shown in (a) of Fig. 1(B) of Ref. 15. However, for a given Q_0, there will be a critical d_c below which there will be net accumulation of beads in the channel. In other words, if d is decreased systematically, there will be a sudden drop of $Q(t)$ when d_c is reached. This accumulation of beads will proceed until the whole channel is filled with beads and this is the regime of dense flows.

The process of this dilute-to-dense transition induced by reducing d is recorded in video as shown in Fig. 1(B) of Ref. 15.

Figure 1 shows the d dependence of $\langle Q(t)\rangle$ when d is reduced or increased systematically at a given Q_0. When d is large (point A), $\langle Q(t)\rangle = Q_0$, the flow is dilute. $\langle Q(t)\rangle$ remains practically independent of d when d is larger than a critical size d_c (A to B in Fig. 1). When d_c is reached, the dilute flow turns to dense flow and the flow rate can be reduced instantaneously by several times to drop from Q_0 to Q_d (B to C). After the transition, Q_d decreases monotonically with reducing d (C to D). The flow jams when d is about the size of four-particle diameters, where permanent arching occurs to cause jamming of the flow (D to E).[10] When increasing d from the jammed phase, the flow starts as dense flow, and the rate $\langle Q(t)\rangle$ increases gradually with increasing d until Q_d reaches Q_0 and turns back to dilute flow as shown by triangle points in Fig. 2 (D through C to A). There is no sudden increase of the flow rate at the transition from dense to dilute flow. We have checked that the dense flow rate curve $Q_d(d)$ (the CD part of curve ACD), follows the Beverloo empirical equation $(d - kd_0)^{3/2}$ with $k = 4$[13,14] and is independent of Q_0.

If the experiment of Fig. 1 is repeated with different Q_0, a family of paths similar to ABCD of Fig. 1 will be formed, shown as dotted lines in Fig. 1. An important

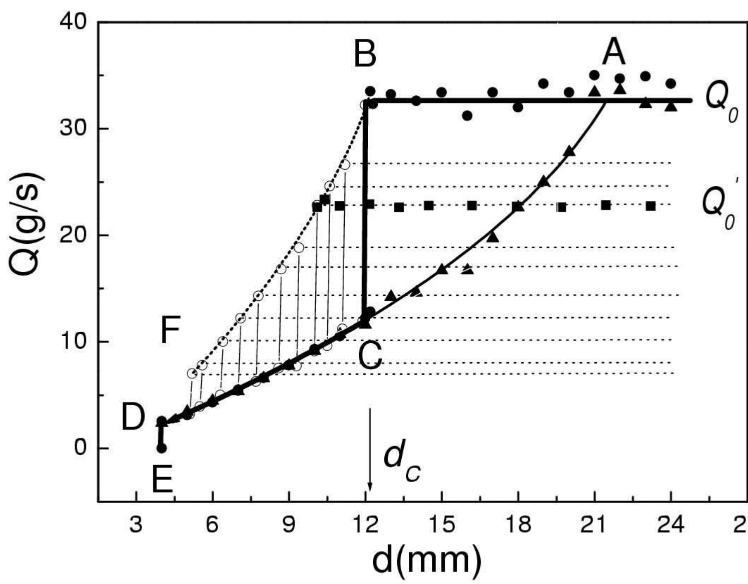

Fig. 1. At a given inflow rate Q_0, the outflow behaves differently when the flow is dilute or dense. For a dilute flow, a transition from dilute to dense occurs at a critical opening size d_c, and the outflow follows curve ABCDE. For a dense flow, as d increases the flow follows curve DC and extended to A with no abrupt change. For different Q_0, the transition occurs at different d as shown by dash lines. The curve BF determines the optimal outflow rate at any given Q_0. For clarity, only two dilute-flow experimental data sets are shown as solid circles and squares.

characteristic of this family of lines is that d_c decreases with Q_0 shown as broken line BF. That is: for a given d, there will be a Q_0 (denoted as $Q_c(d)$ at which a dilute-to-dense transition will occur. Figure 1 is the result of experiments with fixed D but obviously $Q_c(d)$ will also depend on D and d_0. The $Q_c(d)$ curves for various D and d_0 have been measured and shown in Fig. 2. Four D's: $D = 30$, 25, 20 and 15 mm are tested for $d_0 = 1$ mm particles, and two D's: $D = 40$ and 30 mm are tested for 2 mm particles.

In Fig. 2 the upper six curves are $Q_c(d)$'s and lower six curves are the corresponding $Q_d(d)$ curves for particles $d_0 = 1$ mm and 2 mm. If the dilute-to-dense transition scales with the system size, one might expect that the exit width d should be scaled by channel width D as $Q_c(d)/D = F(d/D)$ for some scaling function F. Instead, a new scaling variable $\lambda \equiv \frac{d}{d_0}\frac{d}{D-d}$ is found to collapse the scaled critical flow rate $q_c(\equiv Q_c(d)/(D/d_0))$ into a single scaling curve as shown in Fig. 3. The result in Fig. 3 can therefore be expressed as: $q_c = q_0 F(\lambda)$, where q_0 is some constant.

A remarkable feature of the dilute-to-dense transition is that there are strong fluctuations in $Q(t)$ when d is set to be close to d_c. Direct observations of the motions of the beads close to the exit reveal that there are avalanche-like events taking place

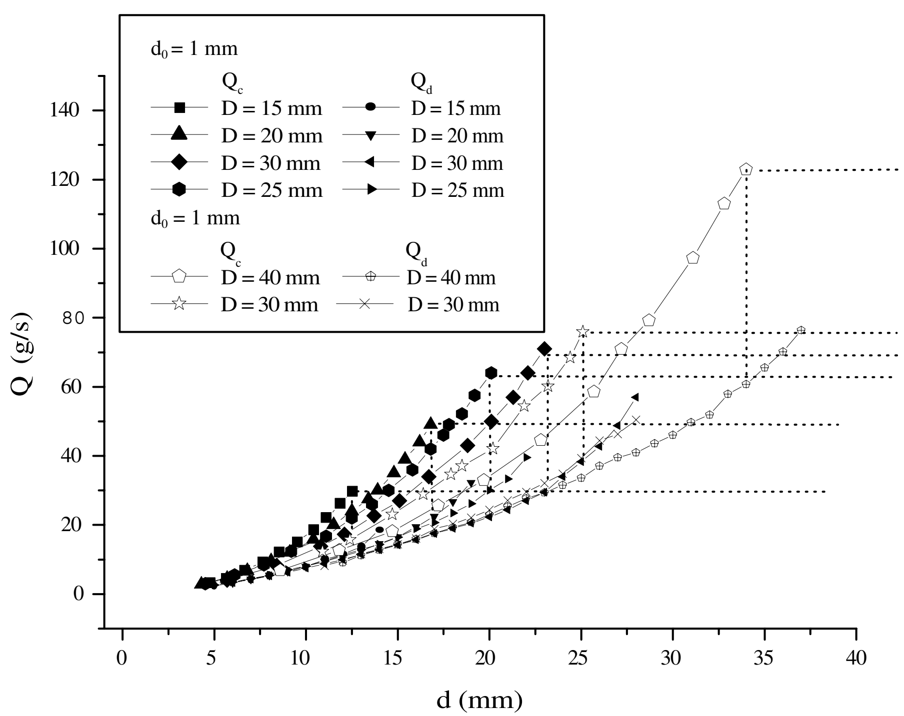

Fig. 2. The Q_c and Q_d versus d of particle size $d_0 = 1$ mm at channel widths $D = 30, 25, 20$ and 15 mm, and of particle size $d_0 = 2$ mm at channel widths $D = 40$ and 30 mm.

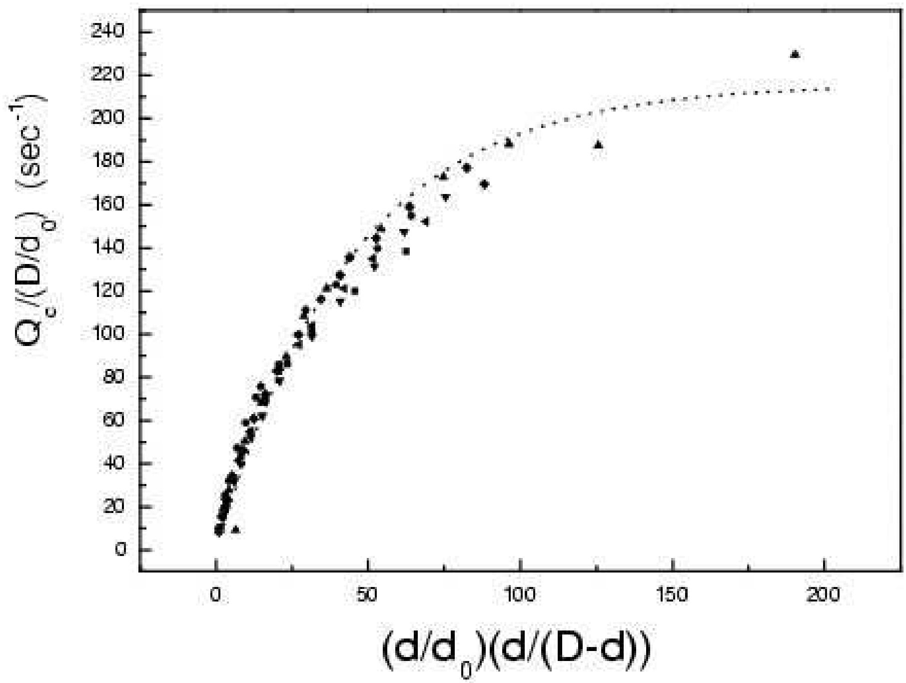

Fig. 3. The scaled $q_c(\equiv Q_c/(D/d_0))$ versus a new scaling variable $\lambda\left(\equiv \frac{d}{d_0}\frac{d}{D-d}\right)$. The dotted line is a fit of q_c by $q_{m(1-e^{-\lambda/\lambda_0})}$, where $q_m = 216$ and $\lambda_0 = 45$.

at the two wedges on either side of the exit. After the wedges have been built up by the incoming flux, flows of surface layer in the form of avalanches will occur. The two wedges effectively act as collectors of incoming flux and direct them to the exit through the surface flow. Therefore, there are strong density fluctuations (in terms of area fraction p) near the exit. When the discharge from the two wedges meet at the exit, the outflow can sometimes be blocked leading to dense flow. This blockage is intermittent if the incoming flux is not large enough and therefore producing strong fluctuations in $Q(t)$. However, if the incoming flux exceeds Q_c, the blockage becomes permanent and there are accumulation of the beads in the channel until the whole section is filled. Particle density ρ of area 40 mm $\times$ 80 mm at the exit is shown by averaging video pictures in Fig. 4. It is found that a dilute-to-dense transition will occur if ρ reaches a critical value ρ_c. In our experiments, it seems that the value of ρ_c corresponds to that of an area fraction of 0.65 ± 03 (see Fig. 4) and is independent of Q_c, d or D.

Following the observations discussed above, the flow across the channel can be divided into three regions; the central part with length d and the two wedges with base length $(D-d)/2$. If v_0 and ρ_0 are the velocity and density of the dilute flow initiated by the hopper just before particles reaching the wedges, the incoming flux on the wedge is $v_0\rho_0\frac{D-d}{2}$. The outgoing flux of the wedge will be carried away by

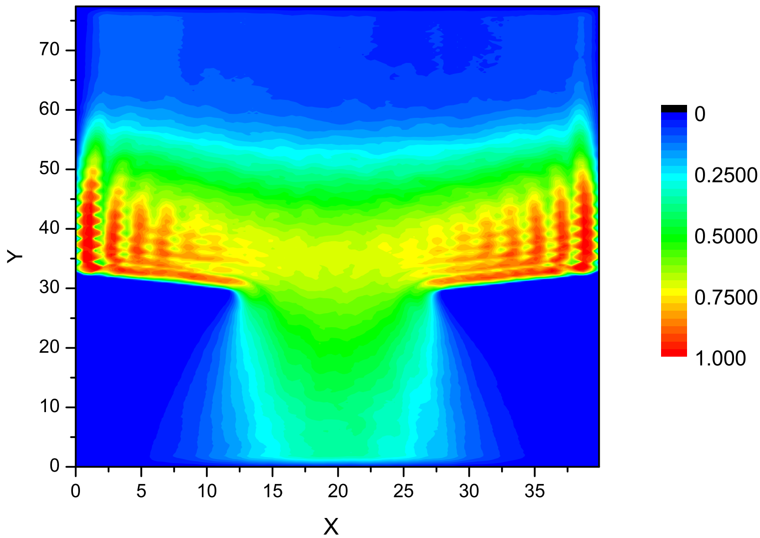

Fig. 4. The average occupancy probability of particles near the exit before transition occurs. In the figure particles flow from top to bottom. The exit width is 16 mm centered at $x = 20$ and $y = 30$.

the fluidized surface layer mentioned as $v_e \rho_c \delta$ where δ and v_e are the depth and characteristic velocity of the surface layer respectively. Conservation of flux gives: $\delta = \frac{v_0 \rho_0}{v_e \rho_c} \frac{D-d}{2}$. In this model, δ will increase with the incoming flux or ρ_0. When these two top layers meet at the exit, we have: $\delta\beta = d/2$ where β is some geometric factor which take care of the angle of repose. Therefore, at the transition, we have:

$$f_c \equiv (v_0 \rho_0)_c = v_e \rho_c \beta^{-1} \frac{d}{D-d} \quad \text{or} \quad \frac{f_c}{f^*} = \frac{v_e}{v^*} \frac{d}{D-d}$$

where $f^* \equiv v^* \rho_c \beta^{-1}$. $\frac{v_e}{v^*}$ is a function of system parameters D and d.

For the case of $D \gg d$, it is reasonable to assume that $\frac{v_e}{v^*}$ depends only on $\frac{d}{d_0}$. It gives $\frac{f_c}{f^*} \sim \lambda \equiv (\frac{d}{d_0}) \frac{d}{D-d}$ which agree with our result in Fig. 4.

In conclusion we have experimentally obtained a dilute-to-dense transition curve $Q_c(Q_0, d_c)$, at which the flow rate Q_0 drops to Q_d at d_c. Instead of the intuitive scaling relationship d/D, experimental results show that a global scaling variable $\lambda \equiv \frac{d}{d_0} \frac{d}{D-d_0}$ determines if the flow is dilute or dense at a given Q_0. The scaled q_c can be fitted in an empirical form $q_m(1 - e^{-\lambda/\lambda_0})$. The ratio λ/λ_0 determines the critical flux of the system. Intuitively, λ_0 is determined by physical properties of the system, such as the elasticity of the particles, particle size and the inclination angle of the plate etc. While the physics of jamming transition is determined by local scales close to d_0, our result suggests that the physics of dilute-to-dense transition is similar to hydrodynamic instability which is controlled by global scales such as $(d/(D-d))$. The discovery of this global scaling property may provide us with ideas of better designs for the transport of granules in industrial processing, and better understanding of similar flow systems such as systems in traffic flows.

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