

Projectile impact and penetration in loose granular bed

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Abstract

Dynamics of a projectile impacting on a loose granular bed under the acceleration due to gravity g has been studied by fast video photograph. Granular jet formation and projectile penetration are observed in 3D and quasi-2D experiments. It is found that the penetration velocity u can be described by $m \frac{du}{dt} = -\gamma u - \kappa z + mg'$, where γ and κ are the parameters which characterize the viscous damping and hydrostatic drag forces of the bed, respectively, z is the penetration distance of the projectile, and g' is a modified gravity term. The viscous damping term is found important in quasi-2D experiments. For 3D, the damping term is only important at the beginning of the impact, and can be neglected during penetration.

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1. Introduction

Granular system is considered as an efficient energy dissipation system [1,2]. If a sphere is dropped from a height H under gravity to a box of sand, it is often found that the penetration distance d is much smaller than H , i.e. the dissipation force F_D can far exceed the projectile weight. In the case of a fluid, F_D can be understood from the dynamical properties of the fluid, which are well known. For granular materials, it is only known recently that properties of a granular bed are governed by the force chains [3,4] in the bed. Presumably dynamics of these chains will determine the forces acting on the projectile during penetrations. However, it is not clear whether this dynamics will give rise to fluid-like phenomena such as viscous damping for the granular penetrations. Recently, there are investigations carried out by Ciamarra et al. [5] and Lohse et al. [6,7] to probe the dynamic properties of a granular bed by impact experiments. A remarkable discovery of Ref. [7] is that for projectiles with zero impact velocity ($u_0=0$), F_D can be

described simply by the term κz , where the parameter κ characterizes the force of bed at a depth of z from the surface; very similar to the case of a static bed. With this form of the drag force, it seems that the fluid-like properties of the bed can be neglected.

However, in Ref. [5], the projectile is found to experience a u_0 dependent uniform deceleration during penetration with high impact velocities; suggesting that fluid-like properties might be important. Intuitively, the drag force in Lohse experiments with zero impact velocities will not be sensitive to the fluid-like properties of the bed, while velocity dependent properties of the bed might be dominant in Ref. [5] high impact velocity experiments. These penetration experiments seem to represent the two limiting cases of the same phenomenon. However there might also be fundamental differences in these two experiments due to the different conditions for the beds; namely a three dimensional (3D) bed in Ref. [7] and a quasi-2D bed in Ref. [5].

In this work we study the dynamics of a projectile impacting on a loose granular bed under the acceleration due to gravity g by fast video photograph. The behavior of the bed is found to be similar to a fluid during impact and different from fluid during penetration. The penetration velocity u can be described by $m \frac{du}{dt} = -\gamma u - \kappa z + mg'$, where γ and κ are the parameters which characterize the viscous damping and hydrostatic drag forces of the bed, respectively, and g' is

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a modified gravity term. The viscous damping term is found important in quasi-2D experiments. For 3D experiments this term is only important at the beginning of the impact, and can be neglected during penetration.

2. Experiment

The granular beds in the 3D experiments are prepared by filling a cylindrical container (diameter=19.6 cm) to a height of 25 cm with hollow cenospheres (diameter ≈ 74 – $100 \mu\text{m}$) of density 0.693 g/cm^3 . The beds are always prepared with the following procedure for homogeneity. Particles are first poured into the bed with a sieve (mesh size=0.4 mm) buried at the bottom. The sieve is then retrieved by slowly pulling it up from the bottom. The volume fraction of the bed produced by this procedure

consistently gives a value of about 0.54. Three spheres, one sphere of diameter 2.5 cm with mass 19.4 g (glass) and two spheres of diameter 2 cm with mass 8.7 g and 12.3 g (hollow copper), respectively, are used as projectiles. For penetration experiments, a metal filament (diameter =0.5 mm, length =26 cm, mass =0.4 g) is attached to the projectile to work as an indicator for the position of the projectile during penetrations when the projectile is not visible. The projectile was hung by the end of the filament with a string at various height from the surface of the bed. The impact experiment is initiated by burning the string. Motions of the projectile are then recorded by a 1000 frames/sec video camera.

Fig. 1 shows what is observed in typical 3D impacting experiments in time sequence. A steel sphere is dropped from a distance 10 cm above the surface of the granular bed. The sphere touches the surface at a time $t=0 \text{ ms}$.

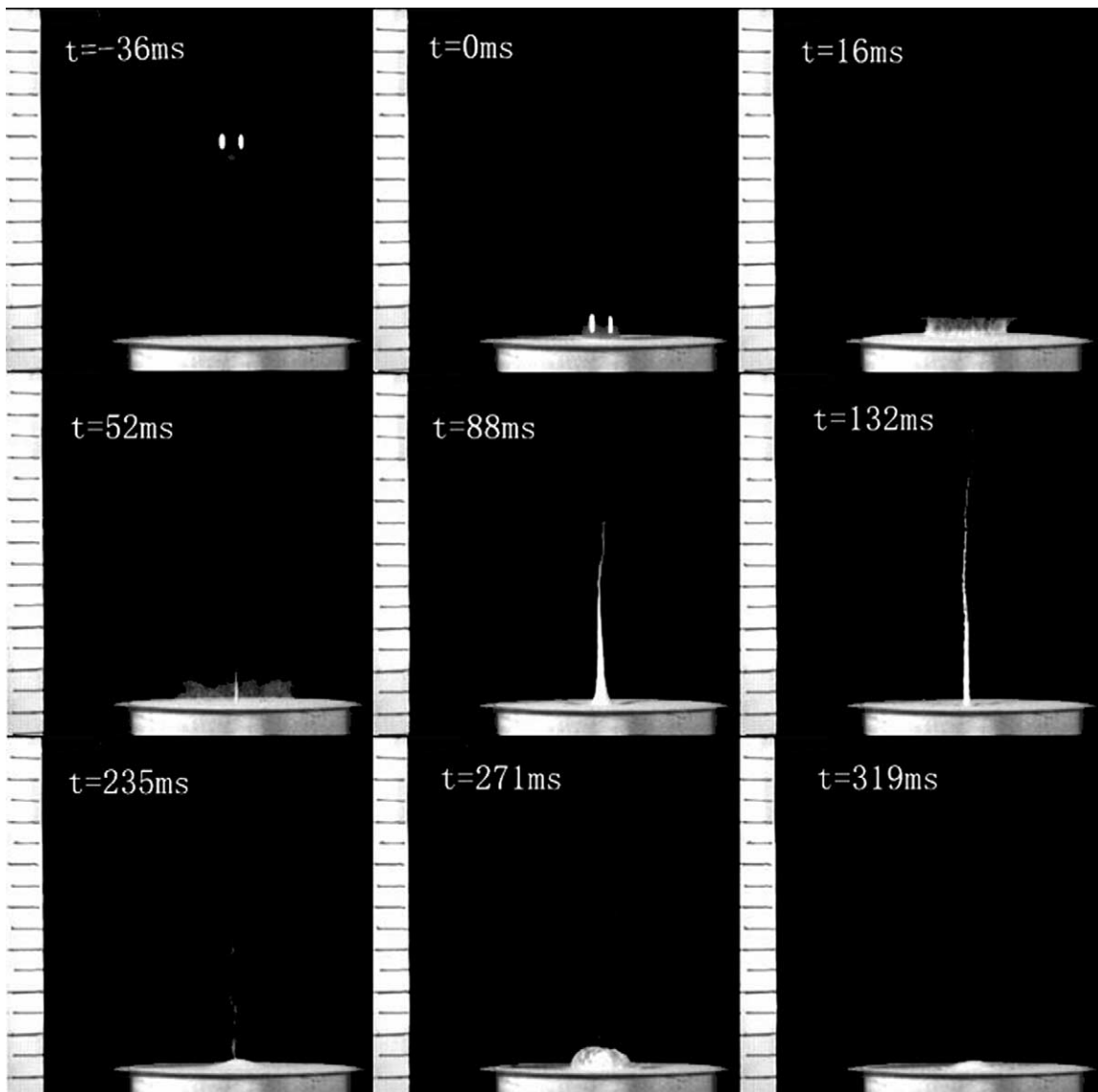


Fig. 1. Jet formation after the impact of a steel ball of $R=5 \text{ mm}$ released from a height of 10 cm on loose fine granules (0.074–0.1 mm).

Granular splash is observed at $t = 16$ ms. A narrow jet then appears at $t = 52$ ms and last for approximately 0.2 seconds. Granular eruption can be observed after the jet vanishes.

In order to see what happens during the penetration of the projectile in the granular bed, we set up a quasi-2D granular bed which has the granular materials confined between a narrow channel. Snapshots of the video are shown in Fig. 2. Let a copper cylinder ($\phi 1.5 \times 1.9$ cm) reaches the surface of the granular bed at a time $t = 0$ ms, a cavity is seen at $t \approx 60$ ms along the penetration and above the cylinder in the bed. It can also be seen that the granules splash similar as the fluid. At $t = 105$ ms, the sidewalls of this cavity collide under hydrostatic pressure at certain point and two jets are

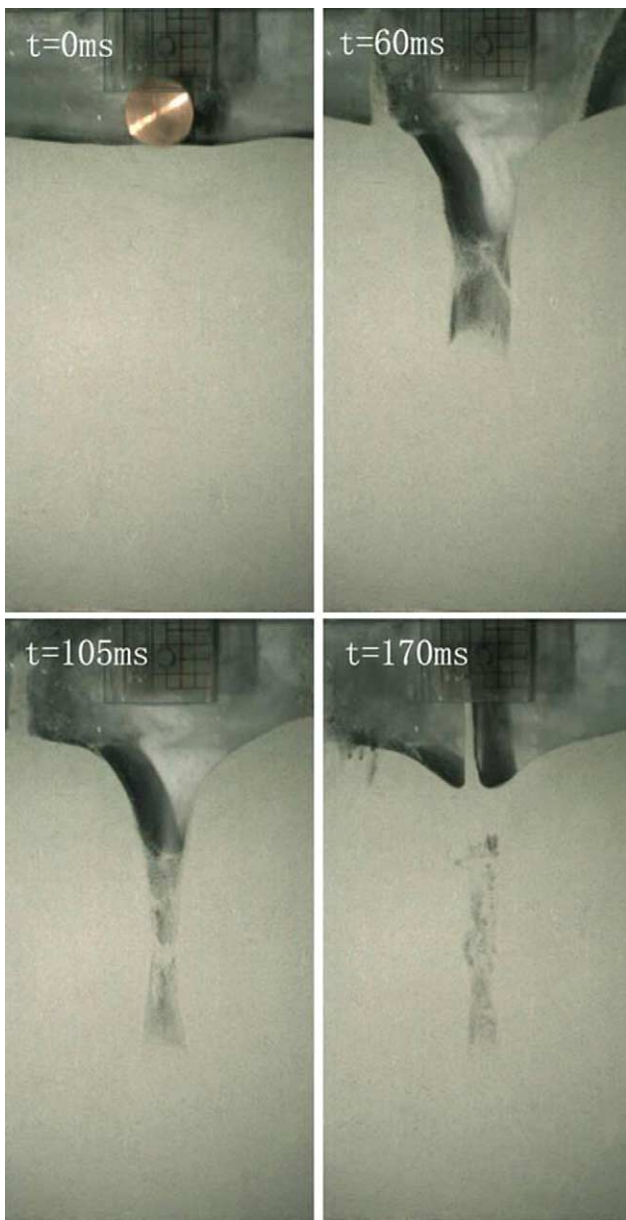


Fig. 2. The penetration of the projectile in the granular bed in time sequence.

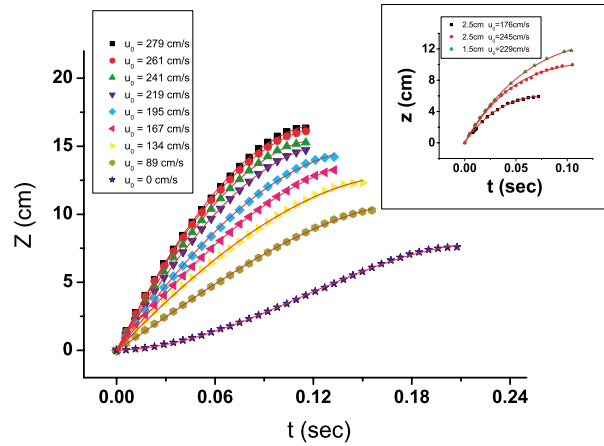


Fig. 3. The time dependence of the penetration depth $z(t)$ of copper sphere ($d = 2.0$ cm and $m = 8.7$ g) with different impact velocities. The inset is the $z(t)$ of copper cylinders of length 1.9 cm and diameter 1.5 cm and 2.5 cm, respectively, measured in quasi-2D experiments.

formed. One goes upward into the air and the other goes downward into the entrained air bubble formed in the void just above the sphere. At $t = 170$ ms, the up going jet reaches the top surface of the bed and can be seen from above the surface.

The time dependence of penetration depth (z) of a copper sphere (diameter $d = 2.0$ cm and mass $m = 8.7$ g) of various impact velocities (u_0) penetrating into the granular bed is measured by using a video camera as shown in Fig. 3. Inset of Fig. 3 shows $z(t)$ of a copper cylinder (diameter $d = 1.5$ cm and 2.5 cm and of mass 28.5 g and 78.7 g, respectively) with impact velocity u_0 penetrating in a quasi-2D bed. Note that u_0 are calculated from the drop height of the spheres and confirmed by measurement from video images just before the entry into the bed. Time dependence of the velocities ($u(t)$) of the projectiles can be determined from Fig. 3 and is shown in Fig. 4. It can be seen that $u(t)$ is not a linear function of t . In fact, except for small u_0 , the general shapes of the curves are convex; $d^2 u/dt^2 < 0$. Since du/dt is proportional to the net damping force acting

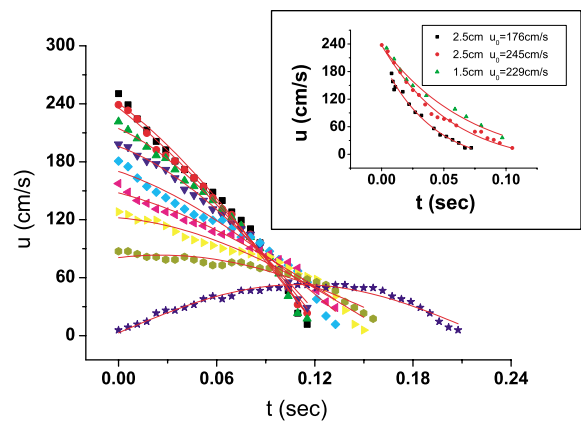


Fig. 4. The time dependence of the velocities ($u(t)$) of the projectile derived from $z(t)$. Inset are curves of $u(t)$ from quasi-2D experiments.

on the projectile, this last finding suggests that the damping force increases with time and therefore with depth z . Results of the time dependence of $u(t)$ in quasi-2D experiments are shown in the inset of Fig. 4. A remarkable feature is that the general shapes of the curves are concave, i.e. $d^2u/dt^2 > 0$; different from those measured in 3D experiments. The difference in the shape of 2D and 3D curves indicates that the mechanism of penetration in the 2D and 3D cases may be quite different.

3. The form of the drag force

From Fig. 2, if we consider the granular bed similar as a fluid, we may write the equation of motion of the projectile as $m \, du/dt = -\gamma u - \kappa z + mg$, where each of these three terms represents the viscous damping, the hydrostatic resistance and gravitational force, respectively. For strong damping, i.e. $\gamma \geq 2\sqrt{m\kappa}$ the solution contains exponential form; for weak damping, $\gamma < 2\sqrt{m\kappa}$ the solution is in sine (cosine) form:

1. Strong damping: $\Gamma = \gamma/\sqrt{m\kappa} > 2$, $z(t/\tau)/z_{\max} = 1 + Ae^{-\lambda' t/\tau} + Be^{-\lambda'' t/\tau}$, where $\tau = \sqrt{m/\kappa}$, $z_{\max} = mg/\kappa$, $2\lambda'' = \Gamma + \sqrt{\Gamma^2 - 4}$ and $2\lambda' = \Gamma - \sqrt{\Gamma^2 - 4}$;
2. Weak damping: $\Gamma < 2$, $z(t/\tau)/z_{\max} = 1 + e^{-\Gamma t/2\tau} A \sin(\omega t/\tau + \phi)$, where ϕ is a phase factor;
3. Critical damping $\Gamma = 2$, $du/dt = -2u/\tau - z/\tau^2 + g$, where $\tau^2 \equiv (m/\kappa)$, $z(t/\tau)/z_{\max} = 1 + e^{-t/2\tau} (A t/\tau + B)$

By fitting the $z(t)$ and $u(t)$ curves in Fig. 3 and 4 (solid lines in Fig. 3 and 4), we find that the 3D results fit to weak damping case and quasi-2D results fit to exponential forms, i.e. critical to strong damping cases, if we let the term g be a fitting parameter $g' = g - f/m$. The physical meaning of the added drag term f is that there is a stress in the bed, which opposes the motion of the projectile during penetration. This force originates from the inelastic collisions between the projectile and the granular materials during the penetration.

For the 3D case, $e^{\Gamma/2\tau} \approx 1$, the fitting parameters are u'_0 , κ , and f . Fig. 5 shows the u_0 dependence of these three fitting parameters. The linear relation between u_0 and u'_0 shows the granular bed can be regarded as a fluid during impact following a modeling of projectile impact on water [8]. If the granular bed is considered as a fluid, the κ term is just the hydrostatic pressure acting on the projectile from the bottom since there is a cavity on top of the projectile during penetration. The coefficient κ can then be expressed as $\kappa \approx \rho_m g \pi r^2$. In our case $r = 1$ cm and $\rho_m = 0.35$, we have $\kappa \approx 1$ N/m, which is about the same order of magnitude as the fitting value of κ shown in Fig. 5. Our fitting value f increases monotonically with u_0 as shown in Fig. 5. Presumably it is due to a more compact bed created by the impact of the projectile when u_0 increases.

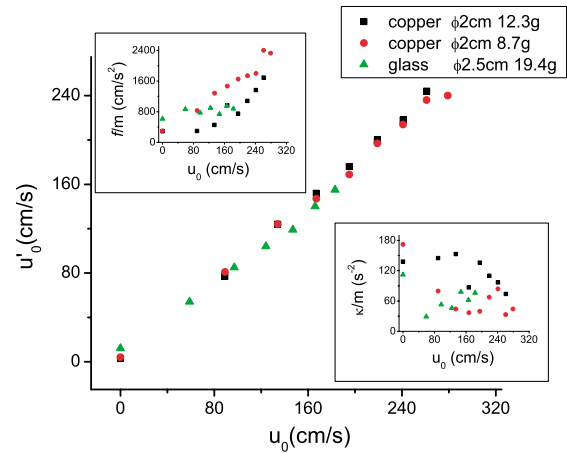


Fig. 5. Fitting parameters u'_0, κ, f as a function of impact velocity u_0 .

4. Conclusion

The picture emerges from discussions above is that there are three main kinds of drag forces acting on the projectile during penetration; namely a viscous damping force γu , a hydrostatic resisting force κz and a friction-like force f . The penetration velocity of a projectile can then be described by equation:

$$m \frac{du}{dt} = -F_D + mg = -\gamma u - \kappa z - f + mg.$$

In the limiting case of a quasi-2D setup, the equation is found to be reduced to

$$m \frac{du}{dt} \approx -\gamma u - f + mg.$$

This viscous damping, however, is only important for early times of 3D penetration. In 3D cases, the equation can be reduced to:

$$m \frac{du}{dt} = -\kappa z - f + mg,$$

similar to the findings of Lohse but with a friction-like force term f . Presumably, this term is related to the granular nature of the fluidized bed but the origin of this f remains to be explored.

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References

- [1] S.T. Thoroddsen, S.T. Thoroddsen, Amy Q. Shen, Granular jets, Phys. Fluids 13 (2001) 4–6.
- [2] J.S. Uehara, M.A. Ambroso, R.P. Ojha, J. Durian, Low-speed impact craters in loose granular media, Phys. Rev. Lett. 90 (2003) 194301.

- [3] D. Howell, R. Behringer, C. Veje, Stress fluctuations in a 2D granular couette experiment: a continuous transition, *Phys. Rev. Lett.* 82 (1999) 5241–5244.
- [4] D. Muth, H. Jaeger, S. Nagel, Force distribution in a granular medium, *Phys. Rev. E* 3164 (1998) 3169.
- [5] M.P. Ciamarra, A.H. Lara, A.T. Lee, D.I. Goldman, I. Vishik, L. Swinney, Dynamics of drag and force distributions for projectile impact in a granular medium, *Phys. Rev. Lett.* 92 (2004) 194301.
- [6] D. Lohse, R. Bergmann, R. Mikkelsen, C. Zeilstra, D. van der Meer, M. Versluis, Ko van der Weele, M. van der Hoef, H. Kuipers, Impact on soft sand: void collapse and jet formation, *Phys. Rev. Lett.* 93 (2004) 198003.
- [7] D. Lohse, R. Rauhe, R. Bergmann, D. van der Meer, Creating a dry variety of quicksand, *Nature* 432 (2004) 689–690.
- [8] E.G. Richardson, E.G. Richardson, The impact of a solid on a liquid surface, *Proc. Phys. Soc.* 352 (1948) 367.