International Journal of Modern Physics B
Vol. 19, Nos. 7, 8 & 9 (2005) 1209–1214

World Scientific
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ELECTRIC FIELD-INDUCED INTERACTION FORCE BETWEEN TWO SPHERES

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An apparatus is developed to study field-induced forces between two spheres. The mutual forces of a couple of identical spheres made of different materials in various media have been measured precisely as the function of inter-spherical spacing, field strength and field frequency. The respective dependences of the forces on inter-spherical spacing, field strength, field frequency and sphre size are obtained. By comparing the measured results with available theoretical calculations we conclude that a further improvement on the theoretical model should be carried out for explaining the most experimental results when the particle nearly contact.

1. Introduction

Based on the dipole approximation, the interaction force f of two identical homogeneous spheres of radius R in an electric field E_0 can be expressed by [1,2]

$$F_{mutual} = \frac{12\pi\varepsilon_0\varepsilon_m R^6}{d^4} E_0^2\beta^2[(3\cos^2\theta - 1)e_r + (\sin 2\theta)e_\theta], \tag{1}$$

where d is distance between two spheres, θ is the angle between the connective line of spheres and the applied field, the square of dielectric mismatch factor $\beta^2 = \frac{(\sigma_p - \sigma_m)^2 + \omega^2\varepsilon_0^2(\varepsilon_p - \varepsilon_m)^2}{(\sigma_p + 2\sigma_m)^2 + \omega^2\varepsilon_0^2(\varepsilon_p + 2\varepsilon_m)^2}$, $\varepsilon_p,\varepsilon_m$ and σ_p,σ_m are dielectric constants and conductivities of the spheres and the mediums respectively, ω is the frequency of the alternative field. Obviously, the force f is dependent on the external electric field (magnitude and frequency), inter-spherical spacing, particle size, dielectric properties of the spheres and the medium around them. In the case for two nearly contacted spheres, the local field E_{loc} in between the spheres should be much higher than the applied field E_0. The effective factors on the interaction force are still not very clear, although lots of studies were carried out in the calculations [3-5]. However this issue is important for understanding the interaction between the particles in electric field, in electrorheological (ER) fluids for instance. In this paper we present the experimental results on the electric field–induced interaction forces of two spheres, which were measured systematically and precisely.

2 Experimental and results

A specially designed apparatus was applied to measure the interaction between two spheres under an external electric field [6,7]. A computer-controlled elevator was used to adjust the spacing between two spheres with an accuracy of 0.001mm, which was detected with the aid of a grating micrometer. A universal movable plate was installed on the elevator stage for controlling the spheres to a position and the orientation precisely in the angle range of θ=0° –55°. The spheres used in our experiments were all millimeter levels in diameters, of which the surfaces were polished carefully in order to avoid any discharge. In the experiments attractive force f between two spheres was measured with an electronic balance (sensitivity is 0.001g). A detailed expression about the experimental arrangement can be found in our previous publication [6,7].

2.1 Dielectric effects of particle phase and medium phase

In order to examine the dependence of the interaction force on the dielectric property of the particles four kinds of spheres with different dielectric properties were studied. They were $SrTiO_3$ crystal (ε_p =294,σ_p□2x10^{-8} sm^{-1}), glassy quartz ($\varepsilon_p \approx 4$), teflon (ε_p =2.6) and gold, representing the high dielectric, low dielectric, insulating and metallic materials, respectively. All of them were carefully made to be spheres of 6.3 mm in diameter with the warp less than 0.005mm. In the experiment two identical spheres were separated with a gap spacing δ=0.05mm and a 150Hz ac electric field was applied. The nitrogen gas was used as the surrounding medium. The relations of the force f versus field strength are plotted in Fig.1. It can be seen that f shows the quadratic dependence on field strength and the spheres with high dielectric constant or high conductivity result in stronger field induced interaction. Almost no interaction between two teflon spheres can be observed within the sensitivity of the measurement. The quadratic dependence is correlated reasonably well to the dielectric prediction, i.e. the value of the mismatch

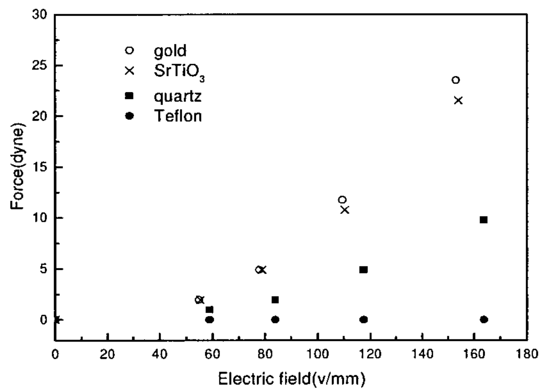

Figure 1 Variation of force f versus field strength of four coordinates: gold-gold, $SrTiO_3$-$SrTiO_3$, quartz-quartz and Teflon-Teflon in nitrogen gas.

factorβplays a pivotal role on the prediction from the dielectric theory.

According to Eq. (1), $f \propto \varepsilon_m \beta^2$. The interaction force of two phases should be affected by the dielectric property of the media around the spheres. Four kinds of media: nitrogen gas (ε_p=1), silicone oil (ε_p =2.54,$\sigma_p \approx 10^{-13}$$sm^{-1}$), castor oil ($\varepsilon_p$ =4.20, $\sigma_p \approx 10^{-13}$$sm^{-1}$) and ethyl benzoate ($\varepsilon_p$ =5.45, σ_p□5x$10^{-8}$$sm^{-1}$) were used in the experiment to measure the interaction force of two $SrTiO_3$ spheres. Because of the high

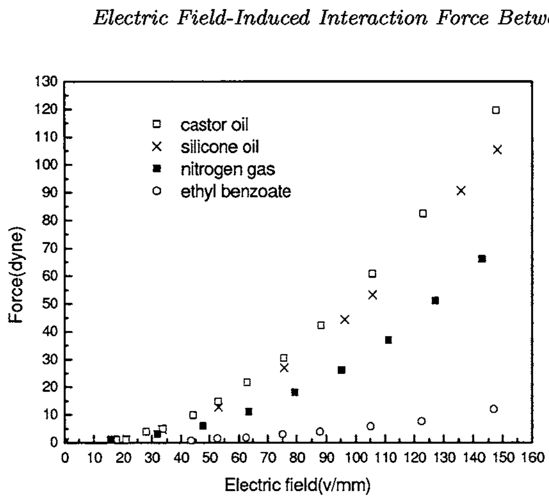

Figure2. Measured forces versus the field strength in four kinds of media, when the spacing is 0.01mm and the frequency is 50Hz.

dielectric constant of $SrTiO_3$ spheres, the square of mismatch factor β^2 tends to be 1 and the variation on the interaction must mainly come from the different dielectric properties of the media. The measured results are shown in Fig.2, where the spacing is 0.01mm and the frequency is 50Hz. It can be seen that the attractive force of two $SrTiO_3$ spheres in castor oil is strongest and then in silicone oil and in N_2, i.e. the sequence of the induced forces is accordant to the relation f $\propto \varepsilon_m$ except in ethyl benzoate. Although the dielectric constant of ethyl benzoate is high, because the conductivity of ethyl benzoate is high (σ_p□$5x10^{-8}sm^{-1}$) and comparable to that of $SrTiO_3$ (σ_p□$2x10^{-8}sm^{-1}$) the force of two spheres is the lowest. This means that the conductivity of the medium play a dominate role on the interaction of particles in a low frequency electric field. It is commonly believed that the response of the ER fluids should increase by using the oil with high ε_f due to $f \propto \varepsilon_f$. However, the high dielectric liquids usually go with high conductivity. It is hard to find the suitable oils in practice to improve the ER response. The ethyl benzoate in our experiments is an example.

2.2 Interspherical spacing and frequency dependence

The spacing dependence of field induced forces between the $SrTiO_3$ spheres is studied in silicone oil as well as in N_2 gas with the gap spacing from 0.01 to 0.8mm. The electric field frequency was fixed to be 50Hz. The variation of the normalized force (f/E_0^2) versus gap spacing δ is shown in Fig.3. By using the dipole approximation and FEA [3-5] the interaction force can be calculated. The values for f/E_0^2 vs. δ are plotted in inset of Fig.3. By comparing the measured results with the calculated ones we can find that the dipole approximation is not valid for the case of the spheres nearly contacted and even the FEA calculation is also not consisted with the measured values whenδ is very small. The increase of the measured force withδ decreasing is much sharper and larger than the calculated ones at smallδ region. This indicates that the available theories for calculating the interaction of particles in the case of their nearly contact should be further improved.

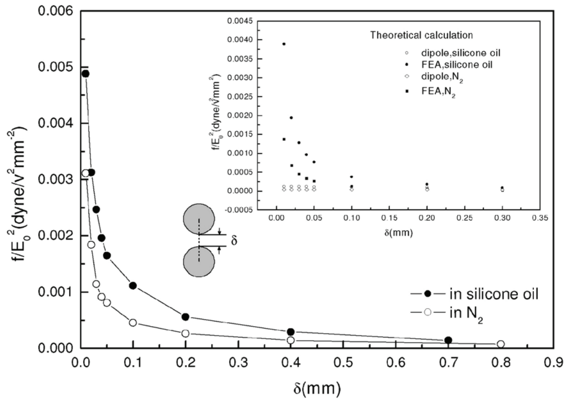

Figure 3. The variation of the normalized force (f/E_0^2) of two $SrTiO_3$ spheres with gap spacing δ. The calculated ones based on dipole approximation and FEA model are given in the inset.

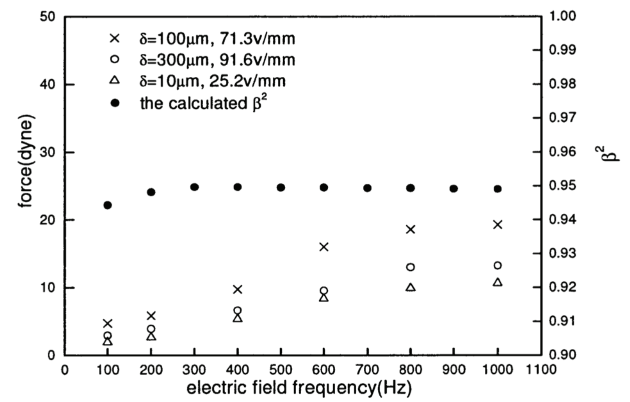

Figure 4. The frequency dependence of the induced force of a couple of $SrTiO_3$ spheres.The calculated β^2 vs. the frequency is plotted with solid circles.

Frequency dependence of inter-particle interaction under an ac electric field has not been well studied, although some experimental data were obtained in ER fluids [8-10]. By employing our apparatus, we have measured the interaction force of two spheres as a function of the field frequency. The frequency dependences of the interaction force between $SrTiO_3$ spheres in silicone oil with δ=0.010mm, 0.100mm and 0.300mm, and at the field strengths of 25.2v/mm, 71.3v/mm and 91.6v/mm were studied, respectively. The measured results are shown in Fig.4. We find that the force between the two spheres, no matter how far they are spaced, increases with the frequency in the range from 100Hz to 1 kHz and the increment is flattened at high frequencies. According to the dielectric model, the mutual force is proportional toβ^2 and the frequency dependence of the force should be same as that of β^2 . However the calculated frequency dependence of β^2 as shown in Fig.4 is not so sensitive on the frequency compared with that of the measured ones. This deviation indicates that the available models are still not consummated for the understanding the interaction between particles when the particles are nearly contacted.

2.3 Particle size effect

Because of $R>>\delta$ in our case, there should be $f \propto R^2$ based on point dipole approximation, while $f \propto R^3$ based on FEA model as two spheres are nearly contacted. In order to know the particle size effect on the induced inter-particle force, we measured the interaction forces for steel spheres with various sizes (R=2.50mm, 2.99mm, 3.15mm and 5.00mm respectively) in a 50Hz electric field. The dependences of the interaction force on the particle size, with the gap spacing δ=0.01mm, 0.05mm and 0.1mm respectively, are shown in Fig.5. Because the measured forces satisfy the relation of $f \propto E_0^2$, so that they can be normalized to be f/E_0^2. By fitting the curves of the f/E_0^2 vs. R a relation of nearly $f/E_0^2 \propto R^3$ can be obtained□which is well consistent with the prediction by the FEA model. These results demonstrate that the particle interaction in an electric field behaves a cubic dependence on the particle size in stead of the quadric dependence as described by dipole approximation.

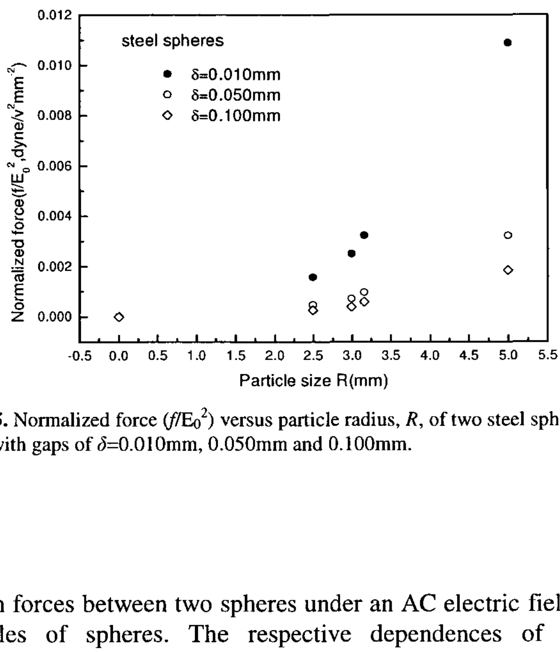

Figure 5. Normalized force (f/E_0^2) versus particle radius, R, of two steel spheres spaced with gaps of δ=0.010mm, 0.050mm and 0.100mm.

3. Conclusions

The interaction forces between two spheres under an AC electric field are measured for different couples of spheres. The respective dependences of the forces on inter-spherical spacing, field strength, field frequency and sphere size are also given. Some important information on the interaction behaviors of the particles in an electric field is provided experimentally. The results demonstrate that the available theoretical calculations are not valid to fit the behaviors of the interaction forces especially for the case of the spheres being closely spaced.

4. Acknowledgements

This work is supported by The Outstanding Overseas Chinese Scholars Fund of Chinese Academy of Sciences and National Natural Foundation of China, No. 10029401. We 'd like to thank Su Zhibin, Men Shouqiang, Lan Yucheng, Xu Xiaoyu , Niu Xiaojuan for large amounts of work in this research and Professor Li Chenxi for helpful discussions.

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