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Criticality of the dilute-to-dense transition in a 2D granular flow

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Abstract
This work investigates the criticality of the dilute-to-dense transition in an inclined quasi-2D granular channel flow. At fixed inflow rate Q_0 and exit opening size d, the waiting time t before the transition occurs after a dilute flow is initiated at $t = 0$ is recorded. The histogram $f(t)$ of the number of times counted that the transition occurs at a time t is plotted as a function of t for each d. It is found that the probability function $C(t)$ for the flow remaining dilute at a waiting time t decays exponentially, and its characteristic time $\alpha^{-1}(d)$ can be fitted well to a power law $a(d_c - d)^{-\gamma}$ with d_c the critical opening size beyond which the transition will never occur.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Properties of granular material are of interest to researchers [1, 2] because of their ubiquity in nature and applications in industry. The nonlinear flow behaviour of granular matter is believed to relate to diverse phenomena such as traffic flow, pedestrian flow, and floating ice [3–5]. These flows present three dynamical 'states': dilute or rapid flow, dense or slow flow, and the jammed state. The flow can be in different 'states' when the density or velocity is changed in various ways. In a dilute flow, particle interactions are dominated by two-body collisions, while in a dense flow, particles interact mainly via multi-body collisions. Due to energy dissipation through inelastic collisions and friction, a dilute (rapid) granular flow will relax to dense flow, and the dense flow to the jammed state [6].

For two-dimensional (2D) channel flows, a maximum inflow rate Q_c is found above which the flow changes from dilute to dense and the outflow rate $\langle Q(t)\rangle$ drops abruptly from the

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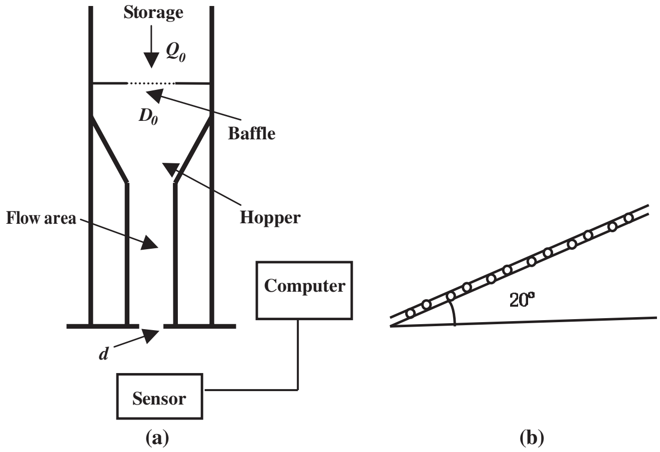

Figure 1. Schematic diagram of the two-dimensional channel. (a) Full-face and (b) lateral views of the experimental set-up.

critical rate Q_c to a dense flow rate Q_d [7]. If we further decrease the exit width, the outflow will eventually be jammed, namely the outflow rate will drop to zero. The dilute-to-dense flow transition is found to depend on the global properties of the flow geometry [7], while the transition from the dense to the jammed 'state' is found to depend only on the ratio of the outlet opening size and the particle size [8]. The criticality of the jamming transition for a finite opening size has been investigated in the 3D silo [9, 10] and 2D hoppers and silos [11]. It is found that the jamming transition is indeed a 'phase' transition, albeit a special one [9, 10].

In reference [7] the critical exit width d_c at which the dilute-to-dense transition of a granular flow occurs in a 2D channel with a confined exit is measured experimentally. This experimentally determined d_c is obtained on the basis of observation during a finite time, which is 15 s for the particular experiment. During this fixed time duration the critical with d_c is determined as the largest d at which the dilute-to-dense transition is observed for a fixed inflow rate Q_0. One may ask the question: if one waits long enough, is this critical opening size still the same under the same experimental conditions, or does it even exist? That is, if one waits long enough, will the transition always occur for any exit width as long as it is smaller than or equal to the opening size of the inflow outlet? If it does, the dilute flow cannot be a steady state. It is only an agitated transient state, which will eventually lose its energy and change to become a dense flow. To answer this question, in this work we experimentally investigate for a 2D granular flow the transition time distribution as a function of the exit opening size to find whether a critical d_c at which the transition time goes to infinity does exist.

2. Experimental set-up and method

The schematic diagram of the experiment set-up is shown in figure 1. The 2D channel is in between a cover plate made of glass and a bottom layer made of a steel plate for eliminating the electrostatic effect. To ensure an almost single-layer flow of steel beads of diameter $d_0 = 2 \pm 0.01$ mm, the gap between the two layers is kept at 2.2 mm. The channel plane is inclined at an angle of 20° with a length of 200 cm. It is divided into two parts; the upper part is the storage for steel beads, and the lower part is the flow section with a width of 60 mm and length of 380 mm.

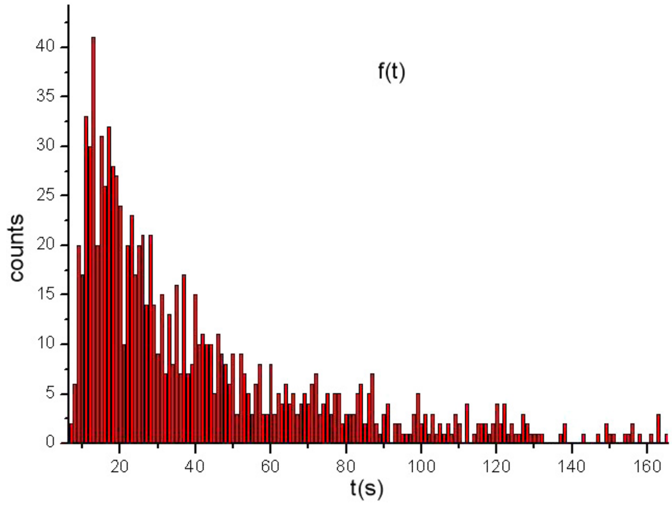

Figure 2. The histogram $f(t)$ of the number of times that the transition occurs at a time t after the flow is initiated at $t = 0$. The result shown is for exit width $d = 19.6$ mm and $N = 1014$.

The inflow rate Q_0 is controlled by the size D_0 of the storage outlet. A typical inflow rate is 34 g s^{-1}. The flow is initiated by pulling out a thin plate placed as a stopper at the exit of the storage hopper. The outflow rate is controlled by the exit opening size d. The exit opening size is controlled by two micrometers to the precision of 0.02 mm at the bottom of the channel. A weighing sensor with the sensitivity of 0.02 g and recording rate of 0.1 s is used to measure the mass of the outflow particles. The slope of the measured mass $M(t)$ is recorded as the outflow rate. At the transition the outflow rate will change abruptly from a dilute flow rate 34 g s^{-1} to a dense flow rate of 21 g s^{-1} in less than one second.

During our measurement the inflow rate Q_0 is kept at a fixed value 34.4 ± 0.4 g s^{-1} ($D_0 = 26$ mm). The time when the flow is initiated is set as $t = 0$. The time t when the transition occurs is recorded for each run. For this inflow rate the transition occurs almost immediately as the flow is initiated when d is smaller than 19.0 mm; when d is greater than 19.7 mm, the transition cannot be observed after a waiting time of 1 h. The measurement is performed at exit size d in the range of 19.0–19.7 mm with an interval of 0.1 mm. For each d the time measurement is repeated about N times, where N is of the order of 10^3.

3. Results and discussion

The histogram $f(t)$ for the number of times that the dilute-to-dense transition is observed at a certain time t was plotted and is shown as figure 2 for $d = 19.6$ mm as an example. After the first few seconds taken for the flow to reach the steady state, the function $f(t)$ decays exponentially.

Let the distribution function $F(t_0)$ be the fraction of counts for all the transitions which occur at a time $t \leqslant t_0$,

$$F(t_0) = \frac{1}{N}\sum_{t=0}^{t=t_0} f(t). \tag{1}$$

We can then define the distribution function $C(t_0)$ that gives the fraction of flows remaining dilute at time t_0, as $C(t_0) \equiv 1 - F(t_0)$. Shown in figure 3 is the function $C(t)$ obtained from

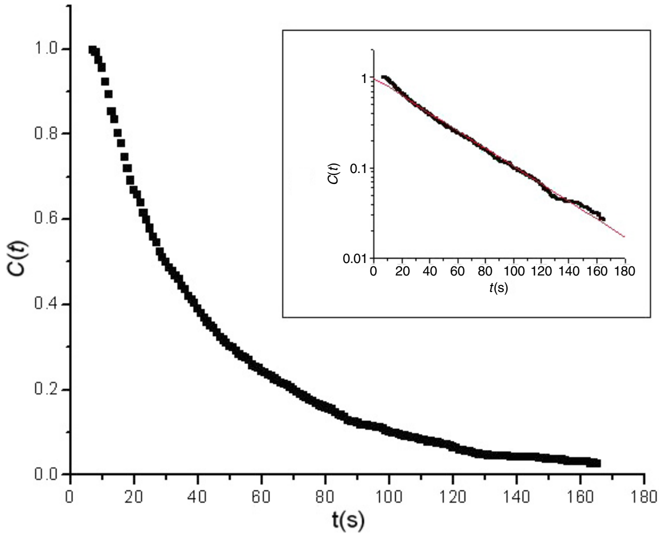

Figure 3. Demonstration of a distribution function $C(t)$ obtained at the exit with $d = 19.6$ mm. The semi-log plot in the inset shows that $C(t)$ fits well to an exponential decay.

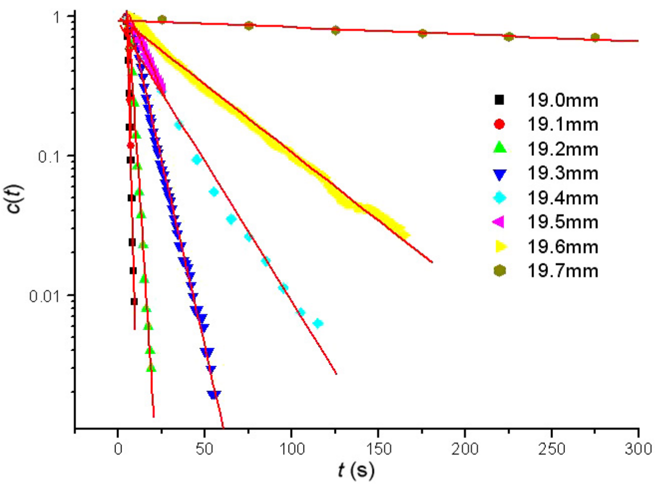

Figure 4. The function $C(t)$ in a semi-log plot at different exit widths from 19.0–19.7 mm. The solid lines are linear fits of the data for each exit width.

the $f(t)$ of figure 2. The semi-log plot of $C(t)$ shown in the inset of figure 3 shows that $C(t)$ is an exponential function of t, i.e.,

$$C(t) = 1 - F(t) = \mathrm{e}^{-\alpha(t-t_1)}, \tag{2}$$

where α^{-1} is the characteristic time for the flow remaining dilute and t_1 is the time for the system to reach a steady flow. Similar $C(t)$ curves obtained at different exit widths d from 19.0 to 19.7 mm also exhibit an exponential form as shown in the semi-log plot in figure 4. The characteristic time α^{-1} can therefore be obtained experimentally. We plot these α^{-1} as a function of d in figure 5. They will fit to an asymptotic form $a(d_c - d)^{-\gamma}$ if a critical exit width

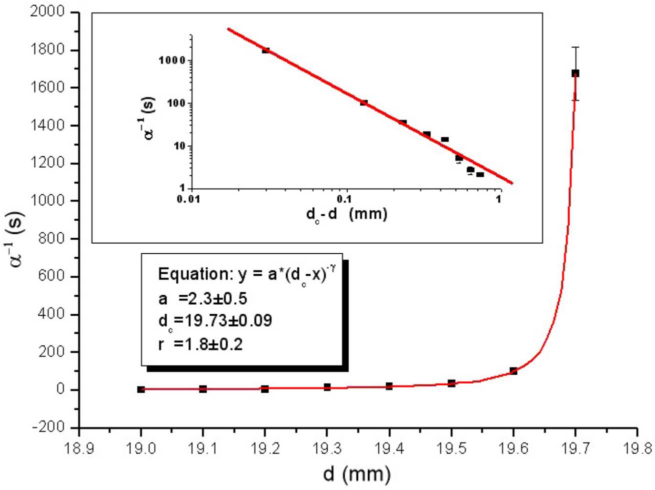

Figure 5. The characteristic time α^{-1} fits well to the power law form $a(d_c - d)^{-\gamma}$ with fitting parameters a, γ and critical exit size d_c. The inset shows the α^{-1} and $(d_c - d)$ power law fitting in a log–log plot. The solid line is drawn to guide the eye.

d_c beyond which the transition will never occur exists. In figure 5 it can be seen that α^{-1} fits well to the form with parameters $a = 2.3 \pm 0.5$, $\gamma = 1.8 \pm 0.2$ and $d_c = 19.73 \pm 0.09$ mm. Using the fitting parameter d_c, we may replot α^{-1} in terms of the parameter $(d_c - d)$ in a log–log plot, as shown in the inset of figure 5. Although we are not able to provide more data points, the log–log plot gives us a good power law dependence. This shows an evidence of the existence of a critical opening size d_c for a given inflow rate and channel width. Beyond this d_c the dilute flow will always remain dilute.

Using the granular diameter d_0 as the characteristic length of the system, we notice that when d is close to d_c, the characteristic time α^{-1} can be increased by two orders of magnitude when the exit is opened by only $0.15d_0$. This shows that the 15 s experimental observation time is sufficient for determining the real value of d_c. In our experiment we also find that the critical exit width d_c ($\sim$19.73 mm) is much smaller than the width of the inflow outlet, which is D_0 (=26 mm). Further experimental study is needed to explore what happens during the transition when the flow rate changes abruptly at constant exit width.

4. Conclusion

In conclusion, we have investigated the criticality of the dilute-to-dense transition by measuring the statistical distribution of the waiting time for the dilute-to-dense transition at a fixed inflow rate and exit width. We find that the probability distribution function for this waiting time decreases exponentially as the time increases. The characteristic time α^{-1} increases as d increases, and it can be fitted well to a power law $a(d_c - d)^{-\gamma}$ with d_c the critical opening size. It is found that the critical opening sizes obtained in experiments with 15 or 1500 s of waiting time differ only within $0.2d_0$, which is presumably due to the fluctuation of the flow density. We can therefore conclude that the dilute state is not a transient state because when d is greater than the critical width d_c, the transition will never occur and the flow will remain dilute.

Acknowledgments

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