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Segregation in Vertically Vibrated Binary Granular Mixtures with Same Size *

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Segregation in vertically vibrated binary granular mixtures with same size is studied experimentally. A new partial segregated state is found in this system. This state exists between the completely mixed state and the pure segregated state, and has the characteristic that the lighter particles tend to rise and form a pure layer on the top of the system while the heavier particles and some of the lighter ones stay at the bottom and form a mixed layer. The ratio of the thickness of the pure top layer and that of the whole system varies continuously with the vibration frequency or amplitude. It is suggested to consider the ratio as an order parameter for describing the degree of the segregation quantitatively. By use of the order parameter, a detailed phase diagram is obtained in Γ *versus* f *space. Finally, the formation of the observed partial segregated state is illustrated by the competition between the impact of momentum of the heavier particles and the stiffness of the layer composed of the mixed particles.*

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Granular materials are ubiquitous and their dynamics are of central importance to many industrial processes. The granular materials can flow like liquids, and there are a variety of theoretical models used to describe such flows.[1-5] These models are referred to as granular hydrodynamics. However, at the present stage the theory is only established for few situations. One example is the so-called granular gas system corresponding to the very dilute and high speed case, which can be described by a kinetic theory similar to that of ordinary gas.[6,7] Another example corresponds to the very dense and low speed case, in which a modified plastic model is adapted[8,9]

One of the most important subjects in the granular hydrodynamics is to explain the segregation phenomena,[4,6] which shows unique mixing and separation behaviour when the material is vibrated[10-16] or flowing.[17-20] The segregation effect mainly results from the difference in particle size, however it also depends on differences of density,[21,22] the properties and angle of repose of the materials,[10,23] the temperature gradient,[24] the total amount of the particles,[25] and even the air in container.[26,27] These seriously disturb the investigation in the segregation. To understand the whole segregation phenomena, one has to study the detailed segregation effect driven by each factor.

The most important size driven segregation is studied by observing the binary mixtures made by the same material but with different sizes (with the air in the container).[25] In their work, Hsiau and Yu[25] have defined a segregation coefficient, which is the relative difference between the number of the larger particles in the upper and lower half of the bed, to quantitatively describe the segregation state. By using the description, they found that the greatest segregation effect occurs when the bed is transformed from a dense state to a loose state, and the segregation coefficient may vary continuously or discontinuously according to the size difference and the total amount of particles. Their work also shows that a quantitative description of the segregation state is crucially important in the investigation of the segregation phenomena. However, the segregation coefficient becomes unclear when the segregation is incomplete, i.e. when the segregation coefficient is between zero and one. In this case, it is hard to image the profile of the distribution of the particles through the segregation coefficient. A similar situation appears in the segregation effect driven by pure density difference. Studies on the binary mixture made by different materials but with the same size (without the air in the container) also found the partial segregated state.[28,29] However, there is no clear quantitative description of the partial segregated state for this system until now. In this Letter, we study the segregation effect in vertically vibrated binary granular mixtures composed of particles with the same size but different densities experimentally, and try to give a clear quantitative description of the partial segregated state occurring in this system.

Our experimental setup consists of a glass cylinder with inner diameter 35 mm and height 150 mm. The bottom of the glass cylinder is made of copper to facilitate conduction of static electricity from the base. This copper base is supported on the horizontal surface of an electromagnetic vibration exciter which

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** To whom correspondence should be addressed. Email: gsun@aphy.iphy.ac.cn moves vertically with a harmonic displacement function $A\sin(2\pi ft)$, where A and f are the amplitude and frequency of the vibration, respectively. In general, frequency f and dimensionless acceleration amplitude $\Gamma = 4\pi^2Af^2/g$ are used as control parameters, where g is the acceleration due to gravity. The vibration exciter is mounted on a horizontally levelled heavy base to ensure the vibration is accurately in vertical. Our experimental apparatus can naturally work in the range of acceleration Γ from 1.0 to 7.0 and frequency f from 18 to 100 Hz. Some previous work has shown that the air in container may play complicated roles in the segregation process.[26,27] To eliminate the influence of air, the container is vacuumized by a mechanical pump to an air pressure less than 50.0 Pa.

Two kinds of granules with different density are prepared by making them with aluminium oxide (1.31 g/cm^3) and cobalt–chromium–molybdenum alloy (8.37 g/cm^3), respectively. The particle size used in experiment is fixed at 0.55 mm. The particles are also selected by rolling them from a slightly inclined plane to ensure that the shape of the particles is close to spherical. The segregation process may also be influenced by the amount of granules in the container or the number of layers of the granules piled in the container.[25] In this Letter, we restrict the amount of granules to 8.0 ml, with 4.0 ml for each component. This amount of granules will pile in about 14 − 16 layers at the bottom of the container.

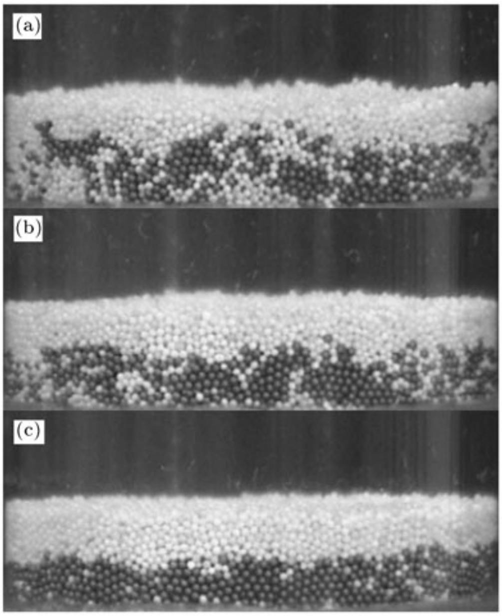

Fig. 1. Photographs for the segregated state of mixture of aluminium oxide and cobalt-chromium-molybdenum alloy particles at frequencies (a) 50 Hz, (b) 60 Hz and (c) 90 Hz, respectively.

Figure 1 presents experimental photographs showing the segregated state at $\Gamma = 6.0$ for different frequencies. From Fig. 1(a), we can clearly see an partial segregated state, which has the character that lighter particles tend to rise and form a pure layer on top of the system, while the heavier ones and some of the lighter ones stay at bottom and form a mixed layer. We call this partial segregated state as a lighter-and-mixed state. Figure 1 also shows that the thickness of the pure top layer increases as the frequency increases, and the partial segregated state is close to the pure segregated state at very high frequency (see Fig. 1(c)), in which the top layer is formed by the pure lighter particles and the bottom layer is formed by the pure heavier ones.

Under the scheme of the lighter-and-mixed state, there is a unique relationship between the component of the lighter particles in the mixed bottom layer and the thickness of the pure top layer if the granular components are equal in volume. Either of them can indicate the degree of segregated state. Thus, we suggest using the ratio of the thickness of the top layer h_1 to that of the whole system h as an order parameter. Apparently, the completely mixed state corresponds to the ratio being zero, and the pure segregated state to it being 1/2. In the lighter-and-mixed state, the ratio is between zero and 1/2, and it describes degree of segregated state quantitatively. The order parameter h_1/h is clearer and more accurate than that of the segregation coefficient defined by Hsiau and Yu,[25] because it comes from a unique description of the segregated state.

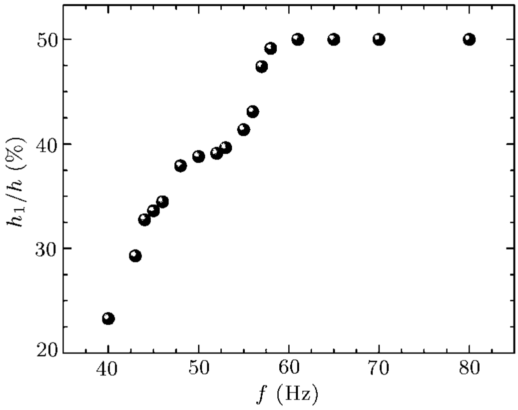

Fig. 2. Experimental results of the order parameter h_1/h as a function of frequency for the mixture of aluminium oxide and cobalt-chromium-molybdenum alloy particles at $\Gamma = 6.0$.

The order parameter is also an observable quantity in experiments. We use the following process to obtain the relative thickness of the top layer. Firstly, we take a digital photograph for the segregated state after the vibration stopped. Then, we delimit the boundary between the top and bottom layers by drawing a lowest line that divided up almost all heavier particles under this line. The determination of the surface and bottom is trivial, and hereafter the calculation of the order parameter from these positions is straightforward. This measurement of the order parameter is fairly stable, except at very low frequency or very high amplitude, where the surface or the boundary between the top and bottom layers becomes uneven. We have checked the measurement of the order parameter by measuring it several times, only little fluctuation is found (less than 5%). It is noticeable that the visible fluctuation is an inherent characteristic of granular material, because of the finite size of the particles. There are some other methods to measure the order parameter, such as, from the average height of each kind of particles as we used in the study by simulations. However, all these methods can not avoid a visible fluctuation except a long-time average is taken.

Figure 2 depicts the experimental results of the order parameter as a function of vibration frequency, which is qualitatively consistent with the property that the system becomes more segregated as the frequency increases (shown in Fig. 1). However, the expression in Fig. 2 contains more detailed information. From Fig. 2, we can find that the order parameter will keep on almost 1/2 for certain range at high frequency, which corresponds to pure segregated state. For very small order parameter, it is difficult to measure the thickness of the top layer experimentally. However, we can approximately obtain a turning point by extending the linear part of the order parameter to the low frequency. Below the turning point, the segregated state corresponds to completely mixed state. The lighter-and-mixed state exists in the crossover range between the two states, where the order parameter is a continuous function as the frequency varies. The saturated characteristic of the order parameter in Fig. 2 is used to define the boundary between the lighter-and-mixed state and the pure segregated state, because there is a turning point between the two states. Similarly, the other turning point at the order parameter near to zero is used to determine the boundary between the lighter-and-mixed state and the completely mixed state.

Under the previous definition, we can obtain the phase diagram in $\Gamma - f$ space (see Fig. 3). In the phase diagram, the pure segregated state appears in high frequency and high acceleration region and the completely mixed state in low frequency and low acceleration region. The lighter-and-mixed state exists in the crossover region between the pure segregated state and completely mixed state at moderate frequency and acceleration. In general, the higher the frequency and larger the acceleration, the better the segregation. There are other two special states in the diagram, i.e. an intense throwing state occurring at low frequency but high acceleration and a weak vibration state occurring at very low acceleration ($\Gamma < 1.2$), which have also been observed by Burtally, King and Swift.[28]

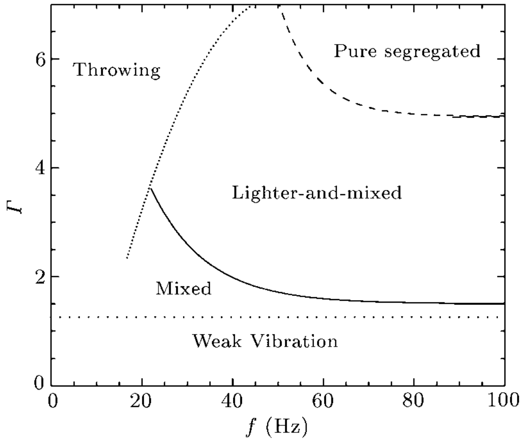

Fig. 3. Schematic phase diagram in Γ versus f space for the mixture of aluminium oxide and cobalt-chromium-molybdenum alloy particles. The boundary between the lighter-and-mixed state and the completely mixed state (the pure segregated state) is shown by the solid (dashed) line. The other boundaries are shown by dotted lines.

We explain the mechanism of the formation of the lighter-and-mixed state as follows. We know that a static layer composed of heavier particles is stiffer than that composed of lighter ones. In other words, when a ball with certain momentum collides with a layer composed of heavier particles, it will sink less than if it collides with a layer composed of lighter ones. In our system, each particle has certain momentum resulting from the vibration, and the heavier particles obviously have more momentum than the lighter ones. On the other hand, the impact to the layer is anisotropic because the boundaries are different. A downward impact is greater than an upward impact, because the boundary at the bottom is closed while that at the top is open. At the beginning, these particles will impact the mixed layer composed of an equal number of both particles. If the stiffness of these equal-component layer cannot support the downward impact of the heavier particles, the heavier particles will sink somewhat and a pure top layer composed of the remaining lighter particles will form. The formation of the top layer of lighter particles will increase the component of the heavier particle in the bottom layer, and hence increase the stiffness of the bottom layer. The heavier particles will stop sinking as the stiffness of the bottom layer and the impact of a heavier particle reach equilibrium. Under this consideration, it can be deduced that a more segregated state should be obtained for the strong vibration acceleration, high vibration frequency, and large density ratio, because all these will enhance the impact of the heavier particles. These properties are qualitatively consistent with what we observed in experiment.

We have experimentally investigated the vertically vibrated binary granular mixtures with the same size but different density. The results show that an partial segregated state, i.e. the lighter-and-mixed state, exists in the moderate strength of vibration (including frequency and amplitude). In the lighter-and-mixed state, lighter particles tend to rise and form a pure layer on the top of the system, while the heavier particles and some of the lighter ones stay at the bottom and form a mixed layer. The ratio of the thickness of the pure top layer to that of the whole system can be defined as an order parameter, which is observable quantity in experiments. We have shown that the order parameter can describe the segregation in more detail. By use of it, a detailed phase diagrams is given, which includes the lighter-and-mixed state in addition to the completely mixed state and the pure segregated state. Finally, the formation of the observed partial segregated state is illustrated by the competition between the impact of momentum of the heavier particles and the stiffness of the layer composed of the mixed particles.

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