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Density-driven segregation in vertically vibrated binary granular mixtures

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Segregation in vertically vibrated binary granular mixtures with the same size is studied experimentally. The partially segregated state occurring in this system is observed carefully. We find that the characteristic of the partially segregated state is that the lighter particles tend to rise and form a pure layer on the top of the system while the heavier particles and some of the lighter ones stay at the bottom and form a mixed layer. The ratio of the thickness of the pure top layer to that of the whole system can be taken as an order parameter, which describes the degree of the segregation quantitatively and is useful in the investigation of the system. By use of it, we find that the segregation state is only dependent on the density ratio of the two kinds of particles. The dependent of the segregation on the vibration frequency is also studied by use of this order parameter, and finally, two typical phase diagrams are given.

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Granular materials are ubiquitous and their dynamics are of primary importance to many industrial processes. The granular materials can flow like liquids, and a variety of theoretical models have been used to describe the flow [1,2]. The investigation of these models is referred to as granular hydrodynamics. However, at the present stage the theory is only established for a few situations. One example is the so-called granular gas system corresponding to the very dilute and high-speed case, which can be described by a kinetic theory similar to that of ordinary gas [3,4]. Another example corresponds to the very dense and low-speed case, for which a modified plastic model is adapted [5,6].

One of the most important subjects in granular hydrodynamics is to explain the segregation phenomenon [1,3], which shows unique mixing and separation behavior when the material is vibrated [7–13] or flowing [14–17]. The segregation mainly results from the difference in particle size; however, it also depends on differences in density [18,19], the properties and angle of repose of the materials [7,20], the temperature gradient [21], the total amount of the particles [22], and even the pressure of air in the container [23,24]. These factors seriously complicated the investigation of the segregation. To understand fully the segregation phenomenon, one has to study the detailed effect of each factor.

The most important size-driven segregation is studied by observing the binary mixture of particles made of the same material but with different sizes (with air in the container) [22]. In the work of Hsiau and Yu, they have introduced a segregation coefficient, which was defined as the relative difference between the numbers of the larger particles in the upper and lower half of the bed, to quantitatively describe the segregation state. By use of the description, they found that the greatest segregation effect occurs when the bed is transformed from a dense state to a loose state, and the segregation coefficient may vary continuously or discontinuously according to the size difference and the total amount of particles. Their work also shows that a quantitative description of the segregation state is crucial in the investigation of the segregation phenomenon. However, the segregation coefficient did not include detailed information on the distribution of the particles unless the segregation is completely segregated or completely mixed. The partially segregated state also appears in the segregation effect driven by a pure density difference [25,26], and we offer a clear quantitative description of the partially segregated state for this system. In this paper, we study the segregation effect in vertically vibrated binary granular mixtures composed of particles with the same size but different densities, experimentally, and try to give a clear quantitative description of the partially segregated state occurring in this system.

Our experimental setup consists of a glass cylinder with inner diameter 35 mm and height 150 mm. The bottom of the glass cylinder is made of copper to facilitate conduction of static electricity from the base. This copper base is supported on the horizontal surface of an electromagnetic vibration exciter which moves vertically with a harmonic displacement function $A\sin(2\pi ft)$, where A and f are the amplitude and frequency of the vibration, respectively. In general, frequency f and dimensionless acceleration amplitude $\Gamma=4\pi^2Af^2/g$ are used as control parameters, where g is the acceleration due to gravity. The vibration exciter is mounted on a horizontally leveled heavy base to ensure that the vibration is accurately vertical. Our experimental apparatus can work in the range of acceleration Γ from 1.0 to 7.0 and range of frequency f from 18 to 100 Hz. Some previous works have shown that the air in the container may play complicated roles in the segregation process [23,24]. To eliminate the influence of the air, the container is evacuated by a mechanical pump to an air pressure less than 50 Pa.

Different densities of granules are realized by making them with different materials. We have prepared many granules with densities ranging from 1.31 to 18.0 $\mathrm{g/cm^3}$ (see Table I). The particle size used in the experiment is fixed at 0.55 mm. The particles are also selected by rolling them from a slightly inclined plane to ensure that the shape of the particles is close to spherical. The segregation process may

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TABLE I. Density of materials used in experiments.

Material	Density $(\mathrm{g/cm^3})$
Aluminum oxide	1.31
Magnesium	1.7
Glass	2.5
Zirconium oxide	2.87
Titanium alloy	4.45
Ferrous alloy	7.44
Cobalt-chromium-molybdenum alloy	8.37
Tungsten alloy	18.0

also be influenced by the amount of granules in the container or the number of layers of the granules piled in the container. In this work, we restrict the amount of granules to 8.0 ml, with 4.0 ml of each component, initially arranged in about 14–16 layers at the bottom of the container.

Figure 1 presents experimental photographs showing the typical segregated states of the mixture of different materials (densities) at $\Gamma=6.0$ and $f=90$ Hz. In these figures, we can see that the states are neither purely segregated nor completely mixed. They are partially segregated. These partially segregated states have the character that lighter particles tend to go up and form a pure layer on top of the system, while the heavier ones and some of the lighter ones stay at the bottom and form a mixed layer. We call this type of partially segregated state a lighter-and-mixed state (LMS). The thickness of the pure top layer increases as the density of the heavier particles increases. For the case of the largest density of heavier particles [Fig. 1(d)], the LMS is very close to the purely segregated state (PSS), in which lighter particles are on top of the heavier ones. For the smallest density of the heavier particles [Fig. 1(a)], it is rather mixed.

Under the previous scheme of the LMS, there is a unique relationship between the component of the lighter particles in the mixed bottom layer and the thickness of the pure top layer if the granular components are equal in volume. Either of them can indicate the degree of the segregated state. Thus, we suggest using twice the ratio of the thickness of the top layer h_1 to that of the whole system h as an order parameter—i.e., $2h_1/h$. Apparently, the completely mixed state (CMS) corresponds to the ratio being 0 and the PSS to it being 1. In the LMS, the ratio is between 0 and 1, and it describes the degree of the segregated state quantitatively. The order parameter $2h_1/h$ comes from a unique description of the segregated state, so it is more clear and accurate than the segregation coefficient defined by Hsiau and Yu [22].

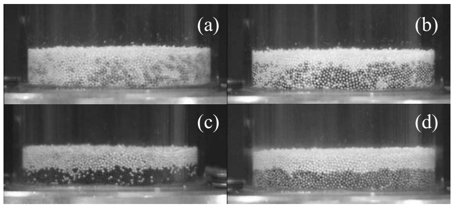

FIG. 1. Photographs of the segregated state of binary granular mixtures. The lighter particles are made of aluminum oxide and the heavier ones are made of (a) zirconium oxide, (b) titanium alloy, (c) cobalt-chromium-molybdenum alloy, and (d) tungsten alloy, respectively. $\Gamma=6.0$ and $f=90$ Hz.

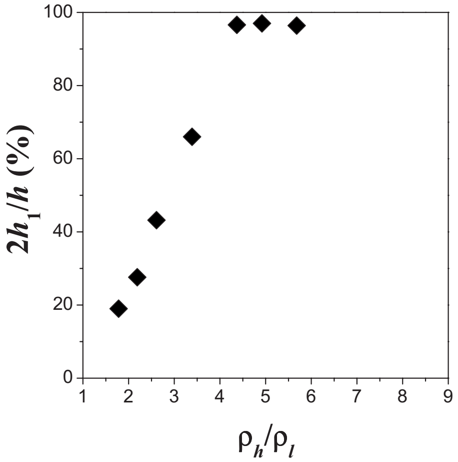

FIG. 2. Order parameter $2h_1/h$ as a function of ρ_h/ρ_l at $\Gamma=6.0$ and $f=90$ Hz. The data from left to right are for mixtures of glass and titanium alloy, aluminum oxide and zirconium oxide, magnesium and titanium alloy, aluminum oxide and titanium alloy, magnesium and ferrous alloy, magnesium and cobalt-chromium-molybdenum alloy, aluminum oxide and ferrous alloy, respectively.

The order parameter is also an observable quantity in experiments. We use the following process to obtain the relative thickness of the top layer. First, we take a digital photograph for the segregated state after the vibration stopped. Then, we delimit the boundary between the top and bottom layers by drawing a lowest line that covers up almost all heavier particles under this line. The determination of the surface and bottom is trivial, and hereafter the calculation of the order parameter from these positions is straightforward. This measurement of the order parameter is fairly stable, except for very low frequency or very large amplitude of vibration, where the surface or the boundary between the top and bottom layers becomes uneven. We have checked the measurement of the order parameter by measuring it several times; only a few fluctuations are found (less than 5%). It is noticeable that the visible fluctuation is an inherent characteristic of granular material because of the finite size of the particles. There are some other methods to measure the order parameter, such as from the average height of each kind of particles. However, all these methods cannot avoid a visible fluctuation unless a long-time average is taken.

Figure 2 shows the experimental results of the order parameter for many combinations of lighter and heavier particles. The data show that all data points collapse into almost one function if the density ratio ρ_h/ρ_l is used as variable, where ρ_h (ρ_l) is the density of the heavier (lighter) particles in the binary mixture. This implies that the order parameter is determined only by the density ratio. The experimental results also show that the order parameter increases monotonically as the density ratio increases and will be saturated at the value of 1 at higher density ratios.

Figure 3 shows two examples of the order parameters as a function of vibration frequency: one is in the region where the system is in an almost PSS at high frequency, and the

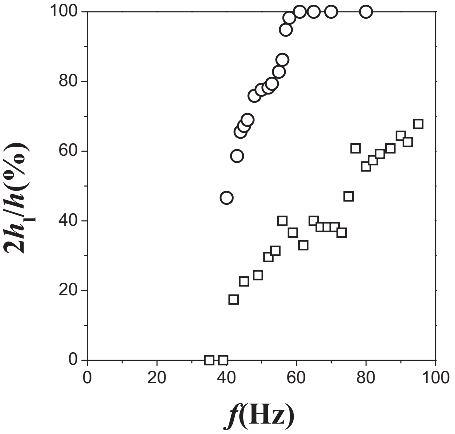

FIG. 3. Order parameter $2h_l/h$ as a function of frequency for two kinds of granular binary mixtures at fixed Γ. The circles are for the mixture of aluminum oxide and cobalt-chromium-molybdenum alloy particles at $\Gamma=6.0$ and the squares for that of aluminum oxide and titanium alloy particles at $\Gamma=4.0$, respectively.

other is in the region where the system is in an almost pure mixed state at low frequency. Both examples show that the order parameter increases as the frequency increases, which means quantitatively that the system becomes more segregated as the frequency increases. In the case of a high density ratio (circles in Fig. 3), the order parameter will remain almost 1 for a certain range at high frequency (over 58 Hz), which can be referred to as the PSS, and turn to an approximate linear variation range at low frequency, which can be referred to as the LMS. In the case of a low density ratio (squares in Fig. 3), the order parameter is almost 0 for certain range at low frequency (below 35 Hz), which can be referred to as the CMS, and turns to an approximately linear variation range at high frequency. The saturated characteristic of the order parameter in Fig. 3 is used to define the boundary between the LMS and PSS, because there is a turning point between the two states. Similarly, the other turning point at the order parameter near zero is used to determine the boundary between the LMS and CMS.

To give a comprehensive understanding of the segregation, we plot a schematic phase diagram in Γ-f space for two kinds of combinations of lighter and heavier particles in Fig. 4, which is determined by definition of the previous paragraph. The solid lines in Fig. 4 show the boundaries either between the LMS and CMS or between the LMS and PSS for the mixture of aluminum oxide and cobalt-chromium-molybdenum alloy particles which has a larger density ratio $\rho_h/\rho_l=6.39$. In Fig. 4, we can see that the PSS appears in the high-frequency and high-acceleration region and the CMS in the low-frequency and low-acceleration region. The LMS can be considered as a crossover region in between the PSS and CMS at moderate frequency and acceleration. In general, the higher the frequency and the larger the acceleration, the better the segregation. As the density ratio varies, the boundaries also change. The dashed lines in Fig. 4 show the boundaries either between the LMS and CMS or between the LMS and PSS for aluminum oxide and titanium alloy particles which has a smaller density ratio $\rho_h/\rho_l=3.40$. We can find that both boundaries shift to high frequency as the density ratio decreases, while they change little with acceleration amplitude. There are other two special states in the diagram; i.e., the intense throwing state occurs at low frequency and high acceleration and the weak vibration state occurs at very low acceleration ($\Gamma<1.2$) [25], the boundaries of both of which are almost unchanged with the density ratio.

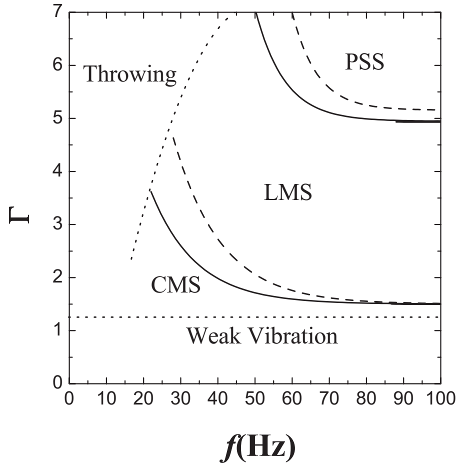

FIG. 4. Schematic phase diagram in Γ versus f space. The boundaries either between the LMS and CMS or between the LMS and PSS for the mixture of aluminum oxide and cobalt-chromium-molybdenum alloy particles (aluminum oxide and titanium alloy particles) are shown by solid (dashed) lines. The other boundaries are shown by dotted lines, which are almost unchanged for different combinations of particles.

In Fig. 5, the schematic phase diagram in Γ-ρ_h/ρ_l space is plotted. The solid line is the boundaries for the fixed frequency $f=90$ Hz. We can easily find that the PSS appears at larger ρ_h/ρ_l and higher Γ and the CMS at smaller ρ_h/ρ_l and lower Γ. Again the LMS occurs in between them as a transition state, and the state at the upper right is segregated better than that at the lower left. When the vibration frequency decreases, as shown by the dashed line in Fig. 5, both boundaries shift to a larger density range, while they change

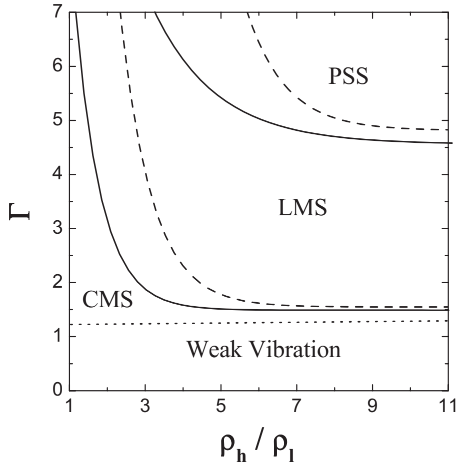

FIG. 5. Schematic phase diagram in Γ versus ρ_h/ρ_l space. The boundaries either between the LMS and CMS or between the LMS and PSS for the vibration frequency $f=90$ Hz ($f=60$ Hz) are shown by solid (dashed) lines. The other boundary is shown by dotted line.

little with acceleration amplitude. In this diagram, the weak vibration state also exists at very low acceleration ($\Gamma < 1.2$), while the intense throwing state does not appear because the frequency is fixed at a relatively high value.

We have experimentally investigated the vertically vibrated binary granular mixtures with the same size but different density. The results show that a partially segregated state exists in the regions of moderate density difference and strength of vibration. In the partially segregated state, lighter particles tend to rise and form a pure layer on the top of the system, while the heavier particles and some of the lighter ones stay at the bottom and form a mixed layer. The thickness of the pure top layer increases as the density ratio increases. The ratio of the thickness of the pure top layer to that of the whole system can be defined as an order parameter, which describes the degree of the segregation quantitatively and can be measured in experiment. We have shown that this order parameter was useful in the investigation of the segregation. As an example, we studied the variation of the order parameter for many combinations of lighter and heavier particles, and found that the order parameter depends only on their density ratio. The dependence of the segregation on the vibration frequency was also studied by use of this order parameter. Finally, the typical schematic phase diagrams in Γ-f space and Γ-ρ_h/ρ_l space are given.

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