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Directed segregation in compartmentalized bi-disperse granular gas*

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A bi-disperse granular gas in an asymmetrical two-compartment system is studied experimentally. The presence of asymmetry within the range of our experimental parameters results in a directed segregated state and a directed clustering state. This deterministic system does not depend on the initial conditions. A modified flux model based on Lohse's flux model for bi-disperse granular gases is derived. The modified flux model explains qualitatively the experimental results.

Keywords: granular matter, segregation, nonlinear dynamics

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1. Introduction

The most prominent feature of the granular gases, which makes them different from the ordinary gases, is the fact that they are dissipative in nature.[1] More frequent collisions of beads occur in a denser region, causing more rapid dissipation of energy and resulting in the formation of clusters of slow moving beads, whereas the relatively dilute regions are depleted and comprised of a few number of fast beads.[2–9] This feature of granular gases can be seen clearly in two-compartment systems. The symmetry breaking phenomenon for the mono-disperse granular gases, called the Maxwell demon, was explained by several models.[10–12] Mikkelsen *et al.*[13,14] investigated the behaviour of a bi-disperse granular mixture of large and small beads and found a tendency of competitive clustering in the compartmentalized system. They explained the phenomenon by applying the generalized Egger's flux model.[10] Due to inelastic collisions and clustering, some interesting states like oscillatory, homogenous, degenerate, etc. have been reported in experiments and simulations.[15–17] The clustering phenomenon also plays an important role in many industrial applications where granular materials require sorting.

Until now, attention has been given to the vertically vibrating compartmentalized systems with mono- and bi-disperse beads, and the compartments have symmetrical structures. In this paper, we will focus on the effect of the asymmetry of the container on the clustering behaviour of a bi-disperse mixture of large and small glass beads in a two-compartment system. We are going to introduce a method for the segregation of a bi-disperse granular mixture by using the asymmetrical two-compartment system.

2. Experimental setup

The experimental setup consists of a container comprised of two compartments, i.e., compartments A and B. The depth of compartment B is kept a constant throughout the experiment, whereas the depth of compartment A can be varied with the help of a slab of adjustable thickness δh. Small glass beads of radius $r_1 = 0.5$ mm and large glass beads of radius $r_2 = 1.5$ mm are placed in the container, which is fixed on a vertically vibrating shaker. The size ratio of the two species is $\psi = 3.0$. The base area of each of the compartments is $\Omega = 25$ mm$\times$25 mm, and the total height of the container is 130 mm. The two compartments of the container are interconnected through

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a rectangular window of area $l \times w = 25$ mm$\times$20 mm, where l and w are the length and the width of the window. In compartment B, the height of the window measured from the base is fixed to be $h = 20$ mm, whereas the height of the window measured from the base in compartment A is varied by using slabs. This container is mounted on an electromagnetic oscillator, which can vertically vibrate the container with sinusoidal oscillations of frequency f and amplitude a, while $\varGamma = a\omega^2/g$ is the driving acceleration, with g being the gravitational acceleration. A schematic diagram of the container is shown in Fig. 1(a).

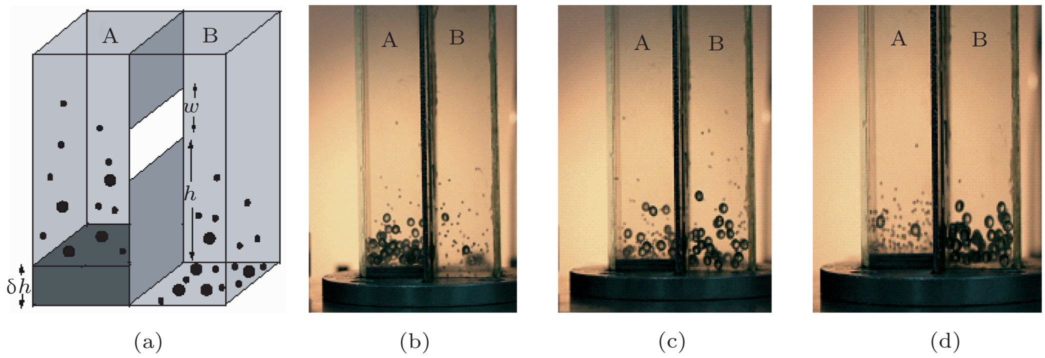

Fig. 1. (colour online) (a) Illustration of experimental setup, where δh is the thickness of slab, h is the height of the window from the base, and w is the width of the window. Panel (b) shows that the small particles, being lighter, move first from compartment A to compartment B. Panel (c) shows that the large particles follow the small particles to compartment B. Panel (d) shows the large particles in compartment B and the small particles in compartment A.

3. Results and discussion

Initially, the two compartments do not have any height difference, and 180 beads in total, $N_{\rm s} = 125$ and $N_{\rm b} = 55$, are placed in compartment A, where $N_{\rm s}$ and $N_{\rm b}$ are the numbers of small and large beads, respectively. The shaking strength is decreased from $\varGamma = 7.5$ g to $\varGamma = 3.0$ g in equal steps of 0.5 g. As the acceleration is decreased, a segregated state is obtained with the large beads in compartment A and the small beads in compartment B. However, initially placing all the beads in compartment B results in a different state, i.e., most of the large beads in compartment B and most of the small beads in compartment A, as shown in Fig. 2(a). Hence, the final state of the system depends on the initial condition of the system. In Figs. 2(a)–2(c), $N_{\rm sB}$(A) and $\rm N_{bB}$(A) are the numbers of small and large beads in compartment B when the system starts with the condition that all the beads initially lie in compartment A, whereas $\rm N_{sB}$(B) and $\rm N_{bB}$(B) are the numbers of small and large beads in compartment B when the system starts with the condition that all the beads initially lie in compartment B. Now increasing the thickness of the slab in compartment A in equal steps of 1 mm, we find that there is again a segregated state at $\delta h = 4$ mm. However, now, the majority of the small beads reside in compartment A, and the large beads stay in compartment B, as shown in Fig. 2(b). The process of segregation is shown in Figs. 1(b)–1(d). By making the system asymmetrical, i.e., different depths, the small beads are always concentrated in compartment A, and the large beads are concentrated in compartment B. It means that the difference of depth in the asymmetrical system directs the majority of the small beads to move in compartment A and the majority of the large beads to compartment B. Such a structure may be the first milestone for further research to develop a system that can be utilized in industry to sort particles.

For slab thickness $\delta h = 14$ mm, most of the small and the large beads are found to dwell in compartment B, and a directed clustering state is found, as shown in Fig. 2(c). In the asymmetrical system, the segregated and the directed clustering states are found to be non-dependent upon the initial conditions. The dependence of the directed segregation on small–large bead ratio $N_{\rm s}/N_{\rm b}$ is investigated for $\varGamma = 6$ at $\delta h = 4$ mm and is plotted in Fig. 2(d). The results show that for $N_{\rm s}/N_{\rm b} = 2$–8, there is a directed segregation with more than 90% of the small beads in compartment A and the large beads in compartment B. For the ratio less than 2, although the number of the large beads in compartment B is small, it still can fluidize the small beads, as a result the small beads are non-uniformly distributed in the two compartments. For a ratio greater than 8, the dissipation due to the large beads in compartment B is so huge that some of the small beads are captured here, and consequently they are non-uniformly distributed in the two compartments.

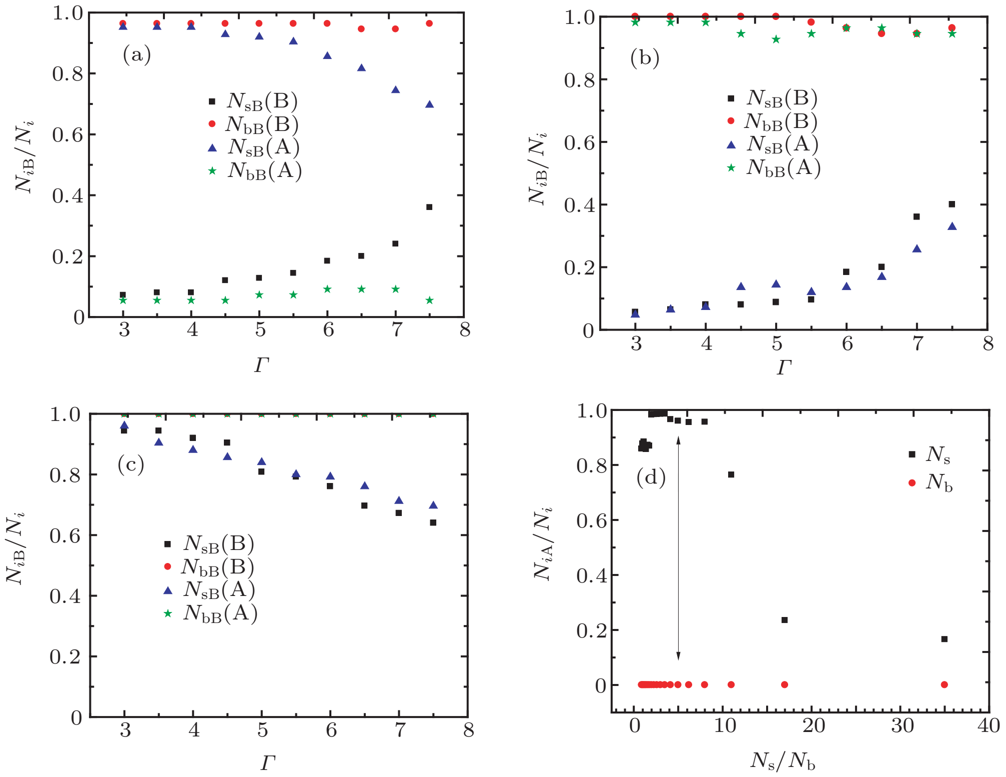

Fig. 2. (colour online) Panels (a)–(c) show the bead ratios in compartment B for $\delta h = 0$, $\delta h = 4$ mm, and $\delta h = 14$ mm, respectively. Panel (d) shows the relation between the small–large bead ratio $N_{\rm s}/N_{\rm b}$ and the bead ratio in compartment A. The directed segregation occurs for $N_{\rm s}/N_{\rm b} = 2$–8 and is indicated by the vertical arrow-head line.

3.1. Flux model

The flux model for bi-disperse granular gases presented by Mikkelsen *et al.*[14] has been utilized to explain our experimental results. A modified flux function for the asymmetrical system is obtained. The flux function is derived based on three main assumptions: i) The temperature of both species is the same; ii) the distribution of temperature for each species is independent of height z; iii) the velocity distribution is Maxwellian and isotropic. So the flux function (see details in Ref. [14]) derived on the basis of these assumptions is given as

$$F_i(N_{\rm s}, N_{\rm b}) = \frac{WN_i}{\varOmega}\sqrt{\frac{k_{\rm B}T}{2\pi m_i}}\,{\rm e}^{-m_igh/k_{\rm B}T}(1-{\rm e}^{-m_igw/k_{\rm B}T}), \quad (1)$$

where m_i and N_i are the mass and the number of either species ($i = {\rm s}$ or b), respectively, $k_{\rm B}$ is the Boltzmann constant, W is the width of the wall, $\varOmega$ is the ground area of the container, and T is the granular temperature. Here the effective temperature is defined by the mean square of the grain velocity fluctuation.

In our case, the flux function for compartment B can be written as

$$F_{i\rm B}(N_{\rm s}, N_{\rm b}) = \frac{WN_i}{\varOmega}\sqrt{\frac{k_{\rm B}T}{2\pi m_i}}\,{\rm e}^{-m_igh/k_{\rm B}T}(1-{\rm e}^{-m_igw/k_{\rm B}T}). \quad (2)$$

Similarly, the flux function for compartment A can be written as

$$F_{i\rm A}(N_{\rm s}, N_{\rm b}) = \frac{WN_i}{\varOmega}\sqrt{\frac{k_{\rm B}T}{2\pi m_i}}\,{\rm e}^{-m_igh'/k_{\rm B}T}(1-{\rm e}^{-m_igw/k_{\rm B}T}), \quad (3)$$

where h' is the height of the window measured from the base in compartment A and

$$h' = h - \delta h.$$

The net flux for compartment B for the two types of beads is

$$\frac{{\rm d}N_{i\rm B}}{{\rm d}t} = -F_{i\rm B}(N_{\rm sB}, N_{\rm bB}) + F_{i\rm A}(N_{\rm sA}, N_{\rm bA}).$$

The conservation of the total number of beads requires that $P_i = N_{i\rm A} + N_{i\rm B}$, so we have

$$\frac{{\rm d}N_{i\rm B}}{{\rm d}t} = -F_{i\rm B}(N_{\rm sB}, N_{\rm bB}) + F_{i\rm A}(P_{\rm s} - N_{\rm sB}, P_{\rm b} - N_{\rm bB}) \quad (4)$$

for compartment B. The net flux for compartment A can be obtained similarly.

Taking the experimental parameters into the flux model, we numerically solve Eq. (4) using the 4th-order Runge–Kutta method. The corresponding flow diagrams for compartment B shown in Fig. 3 enable us to predict how the system evolves. The results show that the modified flux model can predict the directed segregated and the directed clustering states very well. For the value of $\delta h = 0$ mm, there are two fixed points (0.90, 0.37) and (0.10, 0.63), revealing that the final state of the system depends on the initial conditions of the system. The flow diagram shows a segregated state with 90% of the large and 37% of the small beads in one compartment and the rest of the beads in the other compartment, as shown in Fig. 3(a). For the value of $\delta h = 4$ mm, a directed segregated state is obtained, as shown in Fig. 3(b). At first, only the small beads move from compartment A to compartment B. Then, the large beads follow them and occupy compartment B. When the large beads reach compartment B, they fluidize the small beads. Now the small beads ride over the large ones like tennis ball on basket ball and have the probability to cross the window and come back to compartment A. Most of the small beads ride over the large beads, cross the window, and dwell in compartment A. Hence, most of the large beads are found to reside in compartment B, whereas the small beads are found in compartment A, and a directed segregated state is obtained, as shown in Figs. 1(d), 2(b), and 3(b). In the flow diagram, the two fixed points disappear, and there emerges a single fixed point (0.90, 0.37), as shown in Fig. 3(b). It reveals that 90% of the large and 37% of the small beads lie in compartment B. The other beads lie in compartment A. It means that most of the large beads reside in compartment B while the small beads in compartment A. As there is only one stable fixed point, so the directed segregated state does not depend on the initial conditions of the system.

For slab thickness $\delta h = 14$ mm, a directed clustering state is found. In the beginning, the small beads move from compartment A to compartment B. Then the large beads follow them and move to compartment B. As the large beads reach compartment B, they fluidize the small beads. Now the small beads ride over the large ones and have the probability to cross the window and come back to compartment A. So some of the beads, after gaining sufficient energy, come back to compartment A. However, the slab top in compartment A is sufficiently close to the connecting window, which pushes them back to compartment B. Hence, most of the large and the small beads are detained in compartment B, producing the directed clustering state, as shown in Figs. 2(c) and 3(c). There is again a single fixed point (1, 0.64) in the flow diagram. All of the large and 64% of the small beads are found in compartment B, whereas compartment A contains only 36%

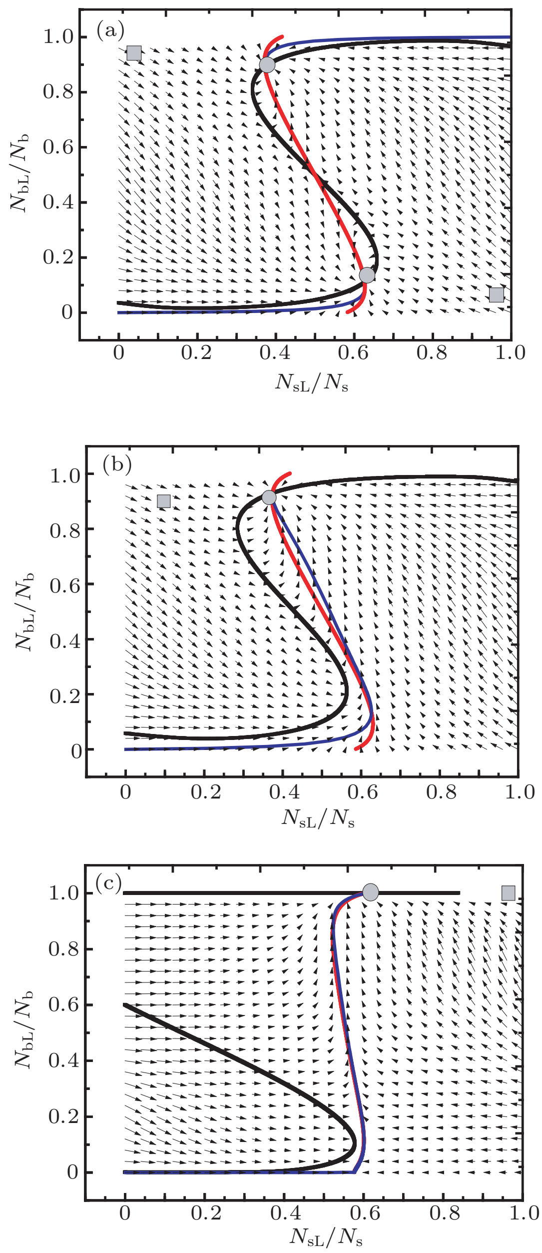

Fig. 3. (colour online) Flow diagrams for (a) $\delta h = 0$, (b) $\delta h = 4$ mm, and (c) $\delta h = 14$ mm, where red and black lines correspond to $\partial N_{\mathrm{s}}/\partial t = 0$ and $\partial N_{\mathrm{b}}/\partial t = 0$ respectively, blue lines show the evolution of the system, squares show the corresponding experimental data, which are (0.95, 0.5) and (0.5, 0.95) for slab thickness 0 mm, (0.9, 0.1) for slab thickness 4 mm, and (1, 0.95) for slab thickness 14 mm.

of the small beads. Thus a directed clustering state, which also does not depend on the initial conditions of the system, is obtained.

Although the theoretical results cannot produce accurate quantitative results due to the simplifications employed, their use lies in elucidating the qualitative trends in the grain behaviour for the asymmetrical system. One possible reason for the inaccuracy is that in Lohse's model, the temperature is assumed to be the same, however the temperature of the two species is not the same.[18] Another reason may be the use of the assumptions of elastic particle–wall collisions and saw tooth base vibrations. The current theoretical results are just able to predict the correct trends. Hence, the theoretical assumptions need to be modified.

4. Conclusion

In this paper, the clustering behaviour of bidisperse granular gas is investigated in a two-compartment system. In one of the compartments, the height of the connecting window from the base is varied in equal steps. The difference of the window heights in the two compartments, within the range of our experimental parameters, results in directed segregated and directed clustering states. These states do not depend on the initial conditions of the system. A modified flux function based on Lohse's flux model for the two-compartment system having asymmetrical column heights is derived. The results obtained from the modified flux model agree with the experimental results qualitatively.

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