Imperfect pitchfork bifurcation in asymmetric two-compartment granular gas

2013 Chinese Phys. B 22 054701

(http://iopscience.iop.org/1674-1056/22/5/054701)

Chin. Phys. B Vol. 22, No. 5 (2013) 054701

RAPID COMMUNICATION

Imperfect pitchfork bifurcation in asymmetric two-compartment granular gas*

Zhang Yin(张 因)[a], Li Yin-Chang(李寅阊)[a], Liu Rui(刘 锐)[a], Cui Fei-Fei(崔非非)[b], Pierre Evesque[c], and Hou Mei-Ying(厚美瑛)[a]†

[a] *Beijing National Laboratory for Condensed Matter Physics and CAS Key Laboratory of Soft Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing* 100190, *China*

[b] *Department of Physics, Beijing Institute of Technology, Beijing* 100081, *China*

[c] *Laboratory MSSMat, UMR* 8579 *CNRS, Ecole Centrale Paris,* 92295 *Chatenay-Malabry, France*

(Received 4 December 2012; revised manuscript received 23 December 2012)

The clustering behavior of a mono-disperse granular gas is experimentally studied in an asymmetric two-compartment setup. Unlike the random clustering in either compartment in the case of symmetric configuration when lowering the shaking strength to below a critical value, the directed clustering is observed, which corresponds to an imperfect pitchfork bifurcation. Numerical solutions of the flux equation using a modified simple flux function show qualitative agreements with the experimental results. The potential application of this asymmetric structure is discussed.

Keywords: compartmentalized granular gases, directed clustering, imperfect pitchfork bifurcation

PACS: 47.20.Ky, 45.70.–n, 05.70.Ln **DOI:** 10.1088/1674-1056/22/5/054701

1. Introduction

Pitchfork bifurcation is a fundamental type of local bifurcation to analyze the changes of states in low-dimensional conditions. In systems with random fluctuations or configuration imperfections, some novel dynamical phenomena, especially in the vicinity of phase transition, such as an imperfect pitchfork bifurcation, would appear. Related observations have been found in liquid helium,[1] the Couette flow,[2] electronic circuits,[3] and optical systems.[4]

In recent years, granular systems have been used as laboratory model systems for studying the physical essence of nonlinear phenomena. Different types of bifurcations have been observed in compartmentalized granular gases.[5,6] For a mono-disperse granular gas, the particles populate equally in the two compartments at a high driving velocity, and cluster in one of the compartments when the driving velocity is low, which is known as the granular Maxwell's demon.[7] The phase transition between clustering in one compartment and equipartition of particles in both compartments can be explained as a pitchfork bifurcation by flux models, which describe the particle flow in between these two compartments.[8] In theory, such a system is usually considered to be symmetric, but in reality, an experimental system can never be perfectly symmetric. Uneven surface of the system or unequal sizes of the compartments can cause the system to be asymmetric, and such factors are usually uncontrollable.

To understand the effect of imperfection factors on the dynamics of particles in this system, a controllable asymmetrical structure is introduced to a two-compartment granular system in this work. The influence on the bifurcation behavior of the system is discussed, and a theoretical description through some frequently-used flux models is given to better understand the phenomenon.

2. Experiments

A schematic plot of our experimental setup is shown in Fig. 1(a), which consists of a glass cell with base size $\Omega = 2.5$ cm $\times$ 5 cm and height 15 cm. The cell filled with $N = 800$ glass beads of radius $r = 0.5$ mm is mounted on a shaker of sinusoidal motion. The cell is separated into two equal compartments by an aluminum plate, which has an opening of 2 cm $(L) \times 2.5$ cm (H). The lower edge of the opening is 2 cm above the base. Different slabs of thickness δh ranging from 6 mm to 12 mm are glued one at a time to the bottom of one of the two compartments. In this way, the depths of the two compartments measured from the opening become asymmetric. We refer the shallow one as compartment A and the deeper one as compartment B. The shaker frequency f is fixed at 40 Hz. Previous investigations[9–11] indicated that the relevant control parameter for such a system is the driving velocity $v_b = af$. In this experiment, the velocity v_b is changed by varying the vibration amplitude a.

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11034010 and 11274354), the Chinese Academy of Sciences "Strategic Priority Research Program - SJ-10" (Grant No. XDA04020200), and the Special Fund for Earthquake Research of China (Grant No. 201208011).

†Corresponding author. E-mail: mayhou@aphy.iphy.ac.cn

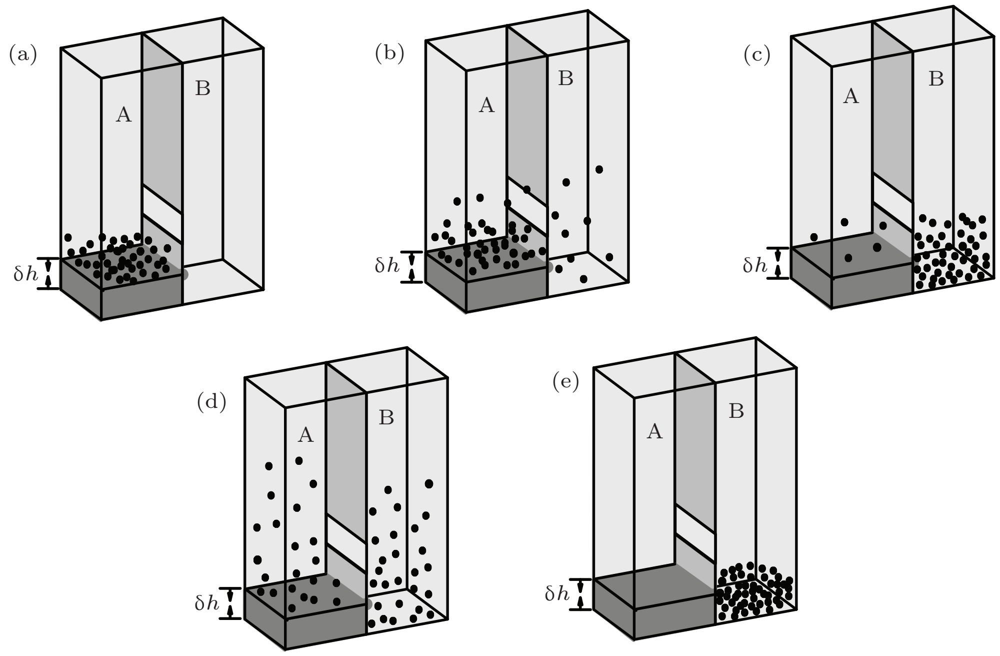

Fig. 1. Illustrations of the system evolution with the initial condition that all particles are located in compartment A. Panel (a) shows the initial distribution, panel (b) shows the distribution when $v_{\rm b}$ is lower than $v_{\rm c}$, panel (c) is the distribution when $v_{\rm b}$ is right at or above $v_{\rm c}$, panel (d) is the asymptotically homogeneous distribution when $v_{\rm b}$ is further increased, and panel (e) is the distribution when $v_{\rm b}$ is lowered again to below $v_{\rm c}$.

By changing the driving velocity, different clustering behaviors are observed, as demonstrated in Fig. 1. The demonstration is under the initial condition that all particles are put in compartment A (Fig. 1(a)) with a slab of thickness $\delta h = 10$ mm. When the driving velocity is lower than a certain critical value $v_{\rm c}$, a small percentage of beads are able to jump through the opening to compartment B, a large fraction of beads remain in compartment A. This is the case shown in Fig. 1(b). While the driving velocity reaches $v_{\rm c}$, nearly all the beads initially in compartment A will cluster in compartment B, as shown in Fig. 1(c). When the driving velocity is higher than $v_{\rm c}$, the beads in compartment B gradually flow back to compartment A, and at a large enough driving velocity the distribution will slowly approach a homogeneous state (Fig. 1(d)). When the driving velocity is reversed to a lower value, the particles initially in compartment A will be relocated to compartment B. Instead of returning to the initial state (Fig. 1(a)) even at the same driving velocity, most of the particles would cluster in the deeper compartment B, as shown in Fig. 1(e). But if we put most of the beads originally in compartment B, no abrupt phase transition would occur. The system would evolve along one phase trajectory without any bifurcation.

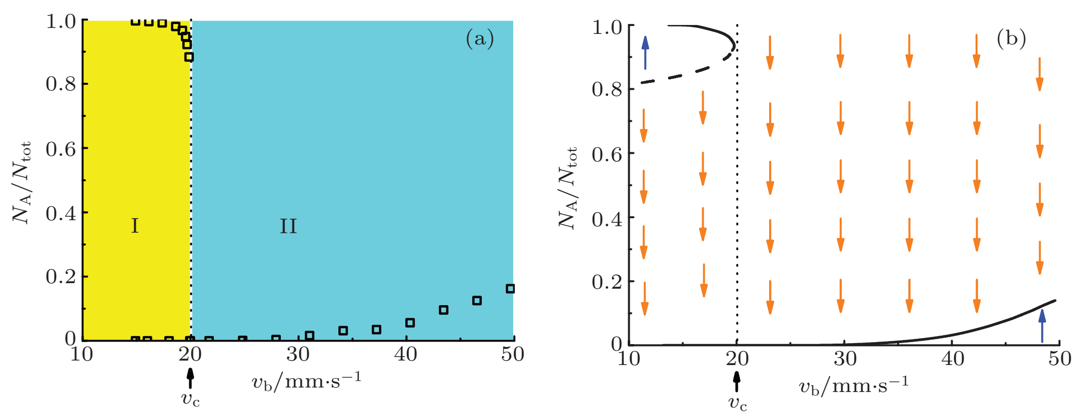

Fig. 2. (color online) When δh is set to 10 mm, the system's evolutions obtained (a) experimentally and (b) theoretically are shown. Particle population of the steady state in compartment A changes with driving velocity $v_{\rm b}$. In panel (b), stable solutions of $\partial N_{\rm A}/\partial t = 0$ are represented by solid lines, while unstable solutions are represented by dashed lines; the arrows of different colors denote the directions of the system evolution under fixed driving velocity, i.e., orange (blue) ones represent $\partial N_{\rm A}/\partial t < 0$ ($\partial N_{\rm A}/\partial t > 0$).

To get a clear picture of the experimental observation, the number of particles in each compartment at each driving velocity is measured (Fig. 2). After a long enough time (usually several minutes after reaching a steady state), the shaker is turned off and the number of particles remaining in each compartment is counted. This procedure is performed several times with different initial conditions at each fixed driving velocity. A phase diagram spanned by N_{A} (the number of particles in compartment A) and v_{b} is obtained. There are two regimes in this phase diagram, as shown in Fig. 2(a). In regime I (below v_{c}), for a low shaking strength, the particles can cluster in either compartment depending on the initial distribution. As the shaking strength is increased, the initial state that most particles are in compartment A will lose stability at a critical velocity v_{c}, and the particles begin to populate in compartment B. When most particles are initially put in compartment B, the particles will evolve continuously without such a sudden change. In regime II (above v_{c}), the system evolves in a relative simple and reversible way. It will approach an asymptotically homogeneous distribution when the shaking strength is increased.

3. Flux model

To understand the behavior of particles in such a system, we have employed a flux model similar to the one used in Refs. [12] and [13], where the system contains a long "staircase" of compartments with a height difference instead of two (in our case). The model describes the time evolution of particle number in each compartment, which is deduced through the local number density and the mean velocity of particles at the height of the opening. In the model, the granular temperature profile is assumed constant, and an isotropic Maxwellian velocity distribution for particles in each compartment is assumed for simplicity. The validity of the assumption of the constant temperature profile along the vibration direction has been investigated by Eggers[7] and more recently by Martin *et al.*[4] and Brey *et al.*[15] The assumption is apparently not valid near the base boundary and at the top of the cell where the Knudsen effect dominates. At the opening position (it is about 20 to 40 particle sizes in height), where the flux is derived, the temperature is roughly a constant.[14] The assumption is approximately held in the region. The assumption that the fluctuating velocities are isotropic at the opening can be justified by comparing the local packing fraction with a critical packing fraction η_r defined in Ref. [14], which is the fraction when the mean time between collisions is equal to the mean time required to cross a horizontal distance equal to the lateral size of the compartment. The packing fraction in our system at the height of the opening is found to be much greater than $\eta_r = \frac{\sqrt{\pi}}{24}\frac{d}{R}$ (here R is the lateral size of the cell and d is the diameter of the particle), which is about 0.003 in our case. Hence sufficient number of collisions near the window between the two compartments can be justified.

An analytical expression of the flux function can therefore be given in the following form:[16,18]

$$F_i = K_i\,\mathrm{e}^{-D_i h_i}(1-\mathrm{e}^{-D_i H}), \tag{1}$$

where K_i and D_i are functions of particle numbers N_i, H is the opening size, and h_i is the height of the bottom edge of the opening in compartment i. We assume that the aluminium bottom plate of the container in our setup is not elastic and the opening size is not small comparing to that of the particle. In our model, we have taken into account of the dissipation due to inelastic collisions between the beads and the bottom plate, and integrated over the window area of the flux. We also assume e_1 and e_2 to be the coefficients of restitution respectively for the particle–bottom and the particle–particle collisions. As the calculation in Refs. [16] and [18], K_i and D_i can be obtained as

$$K_i = \frac{(1+e_1)e_1 N_i v_{\mathrm{b}} L}{(1-e_1^2)\Omega + 4\sqrt{2}\pi(1-e_2^2)N_i r^2},$$

$$D_i = \frac{2[(1-e_1^2)\Omega + 4\sqrt{2}\pi(1-e_2^2)N_i r^2]^2 g}{\pi(1+e_1)^2 e_1^2 v_{\mathrm{b}}^2 \Omega^2}.$$

With the flux function, the evolution of the number of particles in compartment A can be obtained as the net balance between the outgoing flux from A to B and the incoming flux from B to A

$$\frac{\partial N_{\mathrm{A}}}{\partial t} = K_{\mathrm{B}}\,\mathrm{e}^{-D_{\mathrm{B}} h_{\mathrm{B}}}(1-\mathrm{e}^{-D_{\mathrm{B}} H}) - K_{\mathrm{A}}\,\mathrm{e}^{-D_{\mathrm{A}} h_{\mathrm{A}}}(1-\mathrm{e}^{-D_{\mathrm{A}} H}). \tag{2}$$

For a fixed total number N_{tot} $(= N_{\mathrm{A}} + N_{\mathrm{B}})$ in the two compartments, the solution of Eq. (2) is obtained numerically, and the stability of the solution shown in Fig. 2(b) is analyzed by setting $\partial N_{\mathrm{A}}/\partial t = 0$. The solid lines represent stable solutions, which correspond to the steady states of the system. The dashed line represents unstable solutions, which corresponds to the separatrix between these two basins of attraction. The disconnected lower branch arises from a saddle-node bifurcation.

As shown in Fig. 2, the theoretical results show good agreements with the experimental data, which allows us to study the phenomenon beyond the ability of our device. A calculation of the full phase diagram at $\delta h = 7$ mm is shown in Fig. 3. The lower branch of the stable solutions in Fig. 3 can be divided into two phases by the inflection point $(v_{\mathrm{p}}, \lambda_{\mathrm{p}} \equiv N_{\mathrm{Ap}}/N_{\mathrm{tot}})$. When the driving velocity is above v_{p}, the system evolves rapidly to an asymptotically homogeneous state; when lower than v_{p}, the particles will gradually deplete from compartment A.

We can also do the calculation for a very small δh when the corresponding experimental measurements are completely impossible because of large fluctuations. Calculation results for $\delta h = 0$ and very small δh without fluctuations are shown in the insets of Fig. 3 for comparison. In systems with absolute symmetry, we have the usual pitchfork diagram in inset (a). In this case, particles will cluster randomly when the driving velocity is lower than v_c. In a real system with any small asymmetry, the pitchfork becomes disconnected and the bifurcation point breaks up into (v_c, λ_c) and (v_p, λ_p), as shown in inset (b).

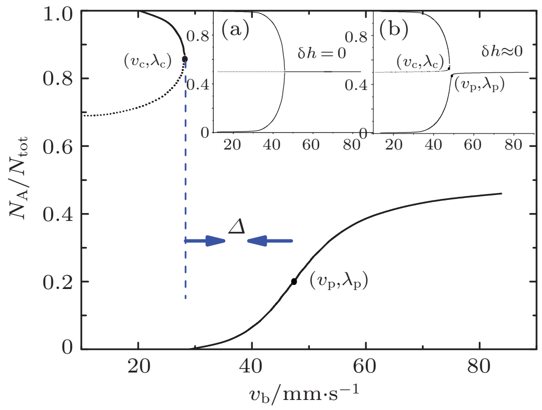

Fig. 3. (color online) Bifurcation diagram when δh is set to 7 mm, where Δ is the range of driving velocity in which the directed clustering occurs. Inset (a) is the symmetric pitchfork bifurcation when δh is zero. Inset (b) shows the condition when a perturbation (very small δh) is added to our system.

The width of the gap from v_p to v_c, $\Delta = (v_p - v_c)$, is a crucial parameter that determines whether the directed clustering will occur. As the gap is small compared to the intrinsic system noise, the particles can still randomly cluster in either compartment at the transition. In other words, the intrinsic noise can switch the system between two branches randomly near the bifurcation point. If we continue to increase value δh so that the gap Δ is large compared to the intrinsic noise, the upper branch is no longer accessible when decreasing the driving velocity, and the fixed point simply glides smoothly along the lower branch. This agrees with the experimentally observed phenomenon of directed clustering.

4. Discussion

As we have seen above, the system evolves from a bistable state to a mono-stable state around certain driving velocity v_c. When changing the controlling parameter δh, the critical velocity v_c will vary accordingly. The dependence of v_c on δh is shown in Fig. 4, where the experimental results are plotted with square dots while the calculation results with a solid line. In the experiments, the opening is 2 cm above the base, therefore the thickness of the slab can be in the range from 6 mm to 12 mm with a step of 1 mm. In this experimental range, v_c decreases monotonically with increasing δh as shown in the figure. Increasing the thickness of the slab, compartment A becomes shallower, which makes it easier for the beads to jump over the barrier to compartment B, therefore reducing the critical velocity v_c. Here $e_1 = 0.89$ and $e_2 = 0.97$ are adopted in the calculation, which are in a reasonable range as given in Ref. [19] for glass beads and an aluminium bottom plate. However, any small variance in these values will not change the curve too much. Hence the agreements shown in the figure indicate that the model has grasped the basic feature of the bifurcation.

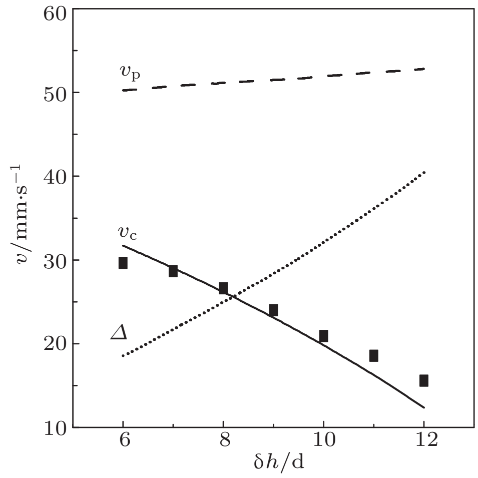

Fig. 4. Dependences of v_c (solid line for theoretical calculation; square dots for experimental results), v_p (dashed line), and Δ (dot line) on δh.

Since Δ is a crucial parameter in determining the occurrence of directed clustering, v_p and Δ in terms of δh are solved numerically, and are shown in Fig. 4. When increasing δh, v_c decreases rapidly while v_p does not change much. The difference Δ increases monotonically with the increasing δh. Therefore δh can be taken as a controlling parameter for the occurrence of directed clustering.

It is interesting to see how the granular gas evolves in compartmentalized systems without gravity. Brey *et al.*[20] investigated the problem in a setup of two connected identical compartments. In that paper, it showed that a spontaneous symmetry breaking is possible in the absence of external fields. A recent work by Isert *et al.*[21] used the properties of diamagnetic particles in strong magnetic field gradients to reduce and even balance the gravitation. A similar symmetry breaking has also been observed in their experiment. In our recent work, a related molecular dynamics simulation has been carried out for an asymmetric system[16,18] under no gravity, and the directed clustering was observed. Although the efficiency is not as significant as that in the case with gravity, the phenomenon does exist, and it is shown that it is possible to improve efficiency by a geometric modification of the configuration, for example, changing the opening position. This may provide a way to manipulate compartmentalized particles under microgravity when a remote control is necessary.

5. Conclusion

In this paper, the directed clustering behavior of a monodisperse granular gas is studied by using a controllable asymmetric configuration in a compartmentalized system. It is found that when the driving velocity is increased from zero, the particles will first cluster in either compartment depending on the initial distribution, and will mostly populate in the deeper compartment when v is above a certain critical value. They will gradually flow back and eventually reach an asymptotically homogeneous distribution as v increases. When evolving in the opposite direction as v decreases, the particles will go from an asymptotically homogeneous distribution to a directed clustering in the deeper compartment. Therefore, the system's evolution depends strongly on its history and the initial distribution. This is different from the behaviors in symmetric systems where the particles homogeneously distribute in both compartments at large v and randomly cluster in either compartment at smaller v. This directed clustering will only occur when Δ is big enough comparing to the system's intrinsic noise. The experimental observation can be well quantitatively described by our modified flux model.

Since the mechanism of clustering is related to the dissipative nature of granular gases, not the gravity, this directed clustering phenomenon in asymmetric configuration may be utilized to provide a means for particle depletion and transporting in a microgravity environment.

References

[1] Schumaker M F and Horsthemke W 1987 *Phys. Rev. A* **36** 354
[2] Abshagen J and Pfister G 2008 *Phys. Rev. E* **78** 046206
[3] Juel A, Darbyshire A G and Mullin T 1997 *Proc. R. Soc. A* **453** 2627
[4] Yabuzaki T, Okamoto T, Kitano M and Ogawa T 1984 *Phys. Rev. A* **29** 1964
[5] Hou M, Li Y, Liu R, Zhang Y and Lu K 2010 *Physica Status Solidi A* **207** 2739
[6] Li Y, Liu R and Hou M 2012 *Phys. Rev. Lett.* **109** 198001
[7] Eggers J 1999 *Phys. Rev. Lett.* **83** 5322
[8] van der Weele K, van der Meer D, Versluis M and Lohse D 2001 *Europhys. Lett.* **53** 328
[9] van der Meer D, van der Weele K and Lohse D 2002 *Phys. Rev. Lett.* **88** 174302
[10] Mikkelsen R, van der Meer D, van der Weele K and Lohse D 2002 *Phys. Rev. Lett.* **89** 214301
[11] Mikkelsen R, van der Meer D, van der Weele K and Lohse D 2004 *Phys. Rev. E* **70** 061307
[12] van der Weele K, Kanellopoulos G, Tsiavos C and van der Meer D 2009 *Phys. Rev. E* **80** 011305
[13] Kanellopoulos G and van der Weele K 2008 *Critical Flow of Granular Matter on a Conveyor Belt* June 29–July 13 2008, Maribor, Slovenia, pp. 112–121
[14] Martin T W, Huntley J M and Wildman R D 2005 *J. Fluid Mech.* **535** 325
[15] Brey J J and Ruiz-Montero M J 2003 *Phys. Rev. E* **67** 021307
[16] Li Y, Hou M and Evesque P 2011 *Journal of Physics: Conference Series* **327** 012034
[17] Hussain Shah S, Li Y, Cui F, Evesque P and Hou M 2012 *Chin. Phys. Lett.* **29** 034501
[18] Hussain Shah S, Li Y, Cui F, Evesque P and Hou M 2010 *Chin. Phys. Lett.* **29** 034501
[19] Kruggel-Emden H, Simsek E, Rickelt S, Wirtz S and Scherer V 2007 *Powder Technol.* **171** 157
[20] Brey J J, Moreno F, Garcya-Rojo R and Ruiz-Montero M J 2001 *Phys. Rev. E* **65** 011305
[21] Isert N, MaaB C C and Aegerter C M 2009 *Eur. Phys. J. E* **28** 205