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Asymmetric local velocity distribution in a driven granular gas

Yanpei Chen
Beijing National Laboratory for Condensed Matter Physics, and Key Laboratory of Soft Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing, China and Lab MSSMat, Ecole Centrale de Paris, Paris, France
Pierre Evesque
Lab MSSMat, Ecole Centrale de Paris, Paris, France, and
Meiying Hou
Key Laboratory of Soft Matter Physics, Institute of Physics, Beijing, China and Beijing National Laboratory for Condensed Matter Physics, and Key Laboratory of Soft Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing, China

Abstract

Purpose – The purpose of this paper is to investigate the local feature of driven granular gases in event-driven molecular dynamic simulation, in order to achieve spatial profiles of local velocity distribution and granular temperature, and the local state with various coefficients of restitution.

Design/methodology/approach – Event-driven molecular dynamic simulation is performed to study a vibro-fluidized granular gas system. Triangular-wave vibration is adopted in the simulation. The authors focus on the steady state of a driven granular gas.

Findings – The simulation finds the local velocity distribution is asymmetric along vibration direction in this driven granular gas system, which agrees with the experimental results obtained in micro-gravity. A nonlinear spatial profile of the skewness of local velocity distribution in vibration direction is found in the simulation. Furthermore, it is found that the value of skewness increases with the system dissipation. It is also found that the two temperature components $T+$ and $T-$ differ from each other. This shows breakdown of energy equipartition. The ratio between them drops exponentially along y direction in various coefficients of restitution. All results confirm that the bulk boundary effect relates to the dissipation properties of granular gases.

Originality/value – This is the first MD simulation that investigates the bulk boundary effect to the local velocity distribution. The spatial profiles of the skewness of local velocity distribution are also investigated when changing the coefficient of restitution to study the influence of the system dissipative nature.

Keywords Granular gases, Local state, Asymmetric velocity distribution

Paper type Research paper

1. Introduction

Dilute granular system, or a granular gas, behaves differently from a classical molecular gas (Grossman *et al.*, 1997; Herbst *et al.*, 2004, 2005; Zippelius, 2006; Baxter and Olafsen, 2003; Losert *et al.*, 1999; Evesque, 2002). Due to inelastic collision,

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energy injection is needed to maintain a steady state. Vibration is a typical agitating mechanism used widely in experiment. This driving mechanism leads a boundary effect which becomes an increasing important issue in the granular gas study.

At first, this boundary heating mechanism introduces anisotropy into a granular gas system because the vibration is usually only in one direction. Anisotropy in such a system has been observed in many experiments and simulations (van der Meer and Reimann, 2006; Yang *et al.*, 2002; Wildman and Huntley, 2000; McNamara and Luding, 1998; Brey *et al.*, 1998; Blair and Kudrolli, 2001, 2003). In addition, most of previous works (Brey *et al.*, 2000; Herbst *et al.*, 2004) assume boundary effect only exists in "the boundary layer", which is true for a classical molecular gas. To avoid this effect, most researchers believe that the central area in the vibration cell is not affected by the boundary heating (Puglisi *et al.*, 2005; Rouyer and Menon, 2000). In fact, theoretical derivation (Esipov and Poschel, 1997) also assume the uniform heating to explain the exponential distribution for a driven granular gas, which is not true for the experiment.

However, in our previous micro-gravity experiments (Chen *et al.*, 2012), the local distribution profiles are found to be with sharp distinctions between the two velocity components, v_y (along vibrating direction) and v_x (perpendicular to vibration direction). While the local distributions of v_x are symmetric, local distributions of v_y are asymmetric. Two peaks appear in the local distribution function near the boundary layers. The value of skewness of f(v_y) is non-zero except for the center point of the box. These phenomena demonstrate that this asymmetry is a long range effect.

In fact, this long range boundary effect was first found in bi-disperse granular systems. For a granular system composing of two species of particles, energy equipartition does not persist (Feitosa and Menon, 2002). Furthermore, this energy equipartition is found to be affected by the heating mechanism. Different species of particles are unequally heated at the boundaries, and this unequal heating determines the level of non-equipartition even in the bulk of the system. The boundary effect is never forgotten even in the limit where heating events are rare compared to collision events (Wang and Menon, 2008).

In a word, the boundary effect is a long range effect for either one single species or granular mixtures. Taking no account of the boundary effect is inappropriate in the boundary heating granular system. What causes this long range boundary effect? Apparently, this is not a classical phenomenon for molecular gases. Considering others' work that this long range effect does not exist in inelastic system with uniform heating (van Zon and MacKintosh, 2004, 2005), we deduce that this long range boundary effect results from a combination of two factors, inelasticity of granular media and the boundary heating mechanism. So it is necessary to investigate how the restitution coefficient impacts on the local distribution for a granular gas. The local velocity distribution needs elaborate investigation.

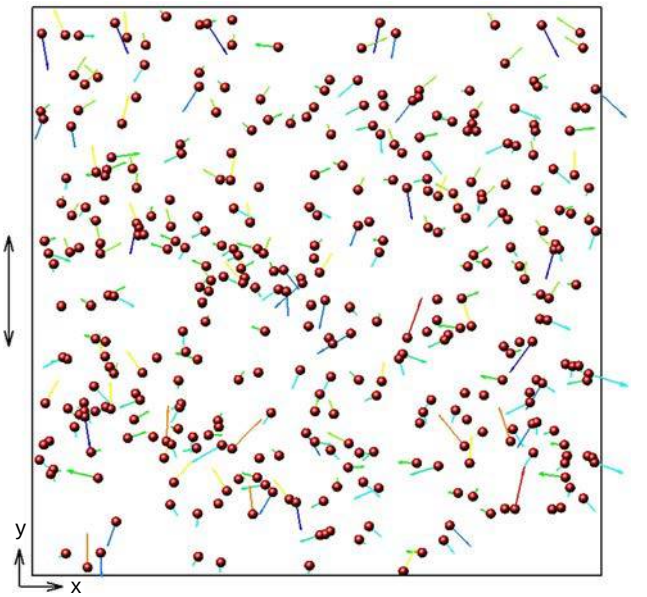
This paper is aimed to investigate local asymmetric velocity distribution with different restitution coefficient using the event-driven molecular simulation. To quantify variation of the local velocity distribution asymmetry, we measure skewness of the local distribution. Our results demonstrate that this long range effect does not exist in elastic system even with the boundary heating. With decreasing value of the restitution coefficient, the local velocity distribution become more asymmetric. Considering the asymmetric distribution, we divide parameters to two parts according to the velocity sign. For instance, the temperature T_y is divided into two components, $T_y^+(y)$ (temperature of particles moving in positive y direction) and $T_y^-(y)$ (temperature of particles moving in the opposite direction). The temperature component $T_y^+(y)$

differs from $T_y^-(y)$. The energy equipartition is broken in our simulation which is consistent with the previous experimental observations (Chen *et al.*, 2012). We also find that the ratio of these two components of temperature $T_y^+(y)$ and $T_y^-(y)$ drop exponentially along y direction. Heat fluxes (q_x^+, q_x^-, q_y^+ and q_y^-) also show anisotropy and a long range effect. All these results confirm that this long range boundary effect depends on the dissipation of a granular gas.

The outline of the paper is as follows. In Section 2, we describe our simulation model of a granular gas. In Section 3, the variations of skewness of local velocity distribution with restitution coefficients are discussed. By using a double-Gaussian model of local velocity distribution, we give skewness expression. In Section 4, spatial temperature profiles are illustrated. In Section 5, we compare the spatial profiles of the heat fluxes for the elastic situation and the inelastic one. In Section 6, we draw our conclusions.

2. The simulation model

We consider a driven granular gas of $N = 360$ inelastic hard spheres with diameter ($d = 2$) and mass ($m = 1$) in a square box ($L = 300$). In this system the area fraction is fixed to be $\phi = 0.0126$ (shown in Figure 1). An event-driven molecular dynamic algorithm is adopted (Poschel and Schwager, 2004). The granular materials are governed by Newton's equation of motion for the center-of-mass coordinates. By solving the differential equations analytically, the trajectories of particles can be computed. This algorithm is called Molecular Dynamics method. During the time intervals between collisions, the particles' motions follow the ballistic trajectories. An assumption used by the event-driven Molecular Dynamics is that there is only one collision occurred at any infinitesimal time duration. Hence, the time-consuming simulation can be replaced by the collision event-consuming simulation. This algorithm



Note: Each vector shows the direction and amplitude of the particle velocity

Figure 1.
The snapshot of granular gases with $N = 360$ by MD simulation

can reduce the useless computation between two successive collisions. No friction and rotation is considered in the model. The vibrating direction is along the y-axis and the boundaries moved in a triangular vibration manner with a peak-peak displacement of $D=5$ and velocity $v_\omega=\pm5$. The particle post-collision velocities with wall are given by:

$$v_y' = -e_\omega v_y + (e_\omega + 1)v_\omega, \tag{1}$$

$$v_x' = -e_\omega v_x, \tag{2}$$

where e_ω is the coefficient of restitution between the particle and the wall. The post-collision velocities of colliding pair, sphere i and j, are determined by:

$$\vec{v}_i' = \vec{v}_i - (1+e_p)\left[\left(\vec{v}_i - \vec{v}_i\right)\cdot\vec{n}\right]\vec{n}, \tag{3}$$

$$\vec{v}_j' = \vec{v}_j + (1+e_p)\left[\left(\vec{v}_i - \vec{v}_j\right)\cdot\vec{n}\right]\vec{n}, \tag{4}$$

where e_p represents coefficient of restitution between the particles, $\vec{n}$ is the unit vector between two mass centers of the collision pair, and e_p and e_ω range between (0,1). Gaussian random number is used for initial velocities and positions of particles. Here, we focus on the "steady" state of the granular system. So, all the particle velocities and positions are recorded after the system go through 10,000 collisions to make sure the system reaches a steady state.

The non-equilibrium thermodynamics is based on local equilibrium hypothesis which means the system is spatially and temporally divided into "cells" or "micro-phases" in small sizes, in which the classical equilibrium theory remains a good approximation. In this paper, we study the local equilibrium by dividing the cell to several bins along the vibrating direction y-axis. Each bin is treated as a local unit and we will examine the local equilibrium state in each bin. In this work the cell is divided into 60 bins. The information of each particle in a bin is gathered. We analyze data in each bin and draw the spatial profile for each parameter (Figure 1).

3. Asymmetric velocity distribution and its skewness

A great number of studies report on the global velocity distribution of a granular gas system. It is known that the velocity distribution in a granular gas driven by boundaries is in the form of stretched exponential, $f(v) = c\exp[-\beta(v/\sigma)^\alpha]$, where $\sigma = <v^2>^{1/2}$, f is the velocity probability distribution function, v is the velocity, and the range of α is found between 1 and 2 (Baxter and Olafsen, 2003; Losert *et al.*, 1999). The global velocity distribution is symmetric. Local distribution is assumed to be a Gaussian distribution considered as a first order approximation in many theoretical works. Rare works examine whether local state of granular media can reach local Gaussian distribution. In fact, the local distribution of velocity component in vibrating direction is asymmetric, while local distribution of velocity component in non-vibrating direction is symmetric. These results are first found in Monte-Carlo

simulation (Brey *et al.*, 2000), then in event-driven molecular dynamics simulation (Herbst *et al.*, 2004) and recently in our experiment in micro-gravity (Chen *et al.*, 2012).

In this section, we provide broader insight into the local velocity distribution in terms of various coefficients of restitution. The spatial profiles of the local velocity distribution of v_x and v_y in a simulation are shown in Figure 2. $f(v_x)$ is found to be symmetric everywhere, while $f(v_y)$ are asymmetric. We also measure skewness of in terms of $f(v_x)$ and $f(v_y)$ under various coefficients of restitution.

Skewness denotes asymmetry of a distribution. The skewness of variable p with n sample points is its third standardized moment, defined as:

$$S = \frac{\frac{1}{n}\sum_{n}^{i=1}\left(p_i - \overline{p}\right)^3}{\left(\frac{1}{n}\sum_{n}^{i=1}(p_i - \overline{p})^2\right)^{\frac{3}{2}}}, \tag{5}$$

here p_i is the variable value of sample point i, $\overline{p}$ is the mean value of all n sample points p_i, and n is total number of sample points. A negative skewness indicates the distribution has a left-side tail, while a positive one indicates a right-side tail.

Figure 3 illustrates spatial profiles of skewness $f(v_y)$ in various restitution coefficients. For $e_p = 0.999$ situation, skewness is almost a constant value, 0. It means the long range boundary effect does not exist even under boundary heating for an elastic granular gas. For coefficients of restitution smaller than 1 (inelastic particle cases), skewness S along y-axis fails to be a constant. Especially, S is the greatest near the boundary and zero at the center. The sign of skewness is positive for the left boundary layer of the cell and negative for the right one. It means the local distribution has a right-side tail in the left boundary, and a left-side fat tail in the right boundary layer. Moving toward the box center, the value of skewness decreases slowly. At the box center, skewness becomes zero. It shows that the boundary effect acts not only at the layer of particles next to the boundary, it extends into the cell, and therefore is a bulk effect, or a long range effect. Moreover, the long effect depends on the inelasticity of the particle. The effect is more profound for a more dissipative (with smaller restitution coefficient) system.

Most of previous studies only focus on a central region in the box in order to avoid the boundary effect. This approach obviously reduces this boundary effect, but is not

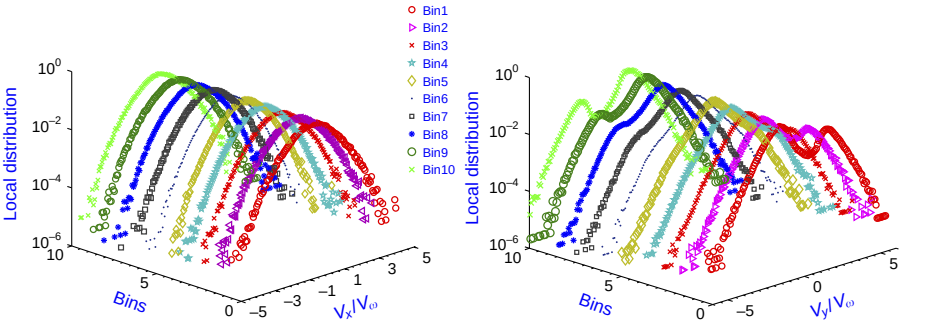
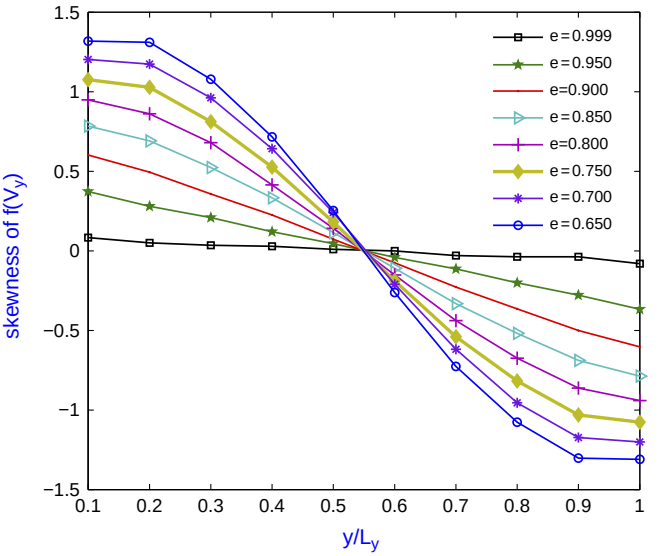


Figure 2.
Spatial profiles of v_x (a) and v_y (b) (semi-logarithmic) are plotted along the y direction

Notes: The particle radius r is set to 1. The size of the cell is $300 \times 300\, r$. The number N of particles is 360, the area fraction is $\varphi = 0.0126$, and the restitution coefficient $e_p = 0.75$



Notes: The coefficient of restitution between particles is from 0.65 to 0.999. The size of the cell is $300 \times 300\,r$, r is the particle radius. The number of particles N is 360, the area fraction $\phi = 0.0126$

Figure 3.
Skewness spatial profiles in various restitution coefficients

able to avoid the boundary effect completely. Because the skewness in central area is smaller than the one the boundary area but is still not zero. It is reasonable that the velocity distribution in the central area is different from the velocity distribution near the wall (Herbst *et al.*, 2005). It is also comprehensible the velocity distribution in the central area is close to symmetric distribution (Herbst *et al.*, 2005). However, it is not proper to overlook this bulk boundary effect for the vibro-fluidized granular system commonly practiced in industries and laboratories.

We expect the coefficient of restitution to have a major impact on the shape of skewness S (shown in Figure 3). Our simulation results show that with higher inelasticity, the variation of skewness becomes larger. The effect not only shows at the boundary bin, but also extends to the whole cell in various coefficients of restitution. Curves of skewness with various coefficients of restitution all pass zero in the box center point. This result is completely different from previous hydrodynamics treatment which assumes the boundary effect only exists in a thin layer (Brey *et al.*, 2000). But here, we find that from the spatial profiles of skewness this boundary effect in vibro-fluidized granular gases can be a bulk effect. This results prove that the classical hydrodynamic does not apply in this case. Comparing with our previous experimental results (Chen *et al.*, 2012), the skewness dependence along y obtained in simulation is not linear curves (shown in Figure 3) as experimentally observed. One possible explanation is that the cell size in unit of particle diameter, L/d, is much larger in simulation than that in the experiment. The ratio is 9 in the Airbus experiment (Chen *et al.*, 2012), but 150 in our simulation here. The simulation result may be more generic.

Figure 4 shows S at the left boundary bin in terms of various coefficients of restitution, which changes nonlinearly and monotonically with changes of restitution coefficient.

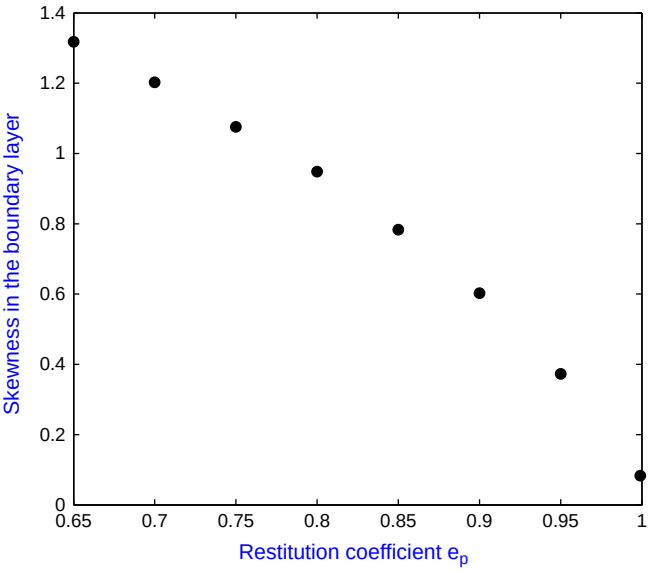


Figure 4.
Skewness in the boundary layer in term of restitution coefficient

Notes: The size of the cell is $300 \times 300\, r$, r is the particle radius. The number of particles N is 360, the area fraction $\phi = 0.012$

For the sake of simplicity, we use a phenomenological model to describe this asymmetric distribution (Chen *et al.*, 2013). The local asymmetric velocity distribution is assumed to be the superposition of two Gaussian functions:

$$f(v_y) = \frac{1}{N}\left(\alpha \exp\frac{(v_y - \xi)^2}{-T} + \exp\frac{(v_y + \alpha\xi)^2}{-T}\right) \tag{6}$$

where parameters (N, ξ, α) in this expression are calculated to make sure the mean velocity is zero, $\int v f(v_y) = 0$. The local velocity distribution is well fitted by this function, as is shown in previous paper (Chen *et al.*, 2013).

By using above local velocity distribution function (the superposition of two Gaussian functions) and the definition of the skewness, we can obtain the skewness S equation:

$$S = \frac{-\alpha\xi^3(\alpha - 1)}{\left(\frac{1}{2}\right)^{\frac{3}{2}}\sqrt{\pi T}(1+\alpha)\left(\mathrm{T} + 2\alpha\xi^2\right)^{\frac{3}{2}}}, \tag{7}$$

From above equation, S is found to be related with parameters α, T and ξ. In the cell center, two Gaussian distributions merge into one, $\xi = 0$. Therefore skewness is equal to zero in the center. Except for this center point, all other points have non-zero S with a value depending on parameters α, T and ξ. The advantage of using a probability distribution function is that S can be obtained with known values of parameters α, T and ξ. However, the shape of local velocity distribution at the boundary layer depends on the vibration mode.

The fat tail of the local velocity profile near the boundary may be explained as follows: the grains can be divided into two types according to its moving direction.

For example, as shown in Figure 1, near the box bottom, the positive velocity particles are those away from the boundary which gain velocity from colliding with the moving wall; while near the upper side, the negative velocity particles are the type gaining velocity and moving away from the wall. While moving away from the boundaries, these two types of particles will gradually lose their velocities through collisions with other bulk particles. The fat tail of the local velocity distribution near the boundary reflects this asymmetry of the positive and negative particle. The fat tail effect will be more profound when the particle restitution coefficient becomes smaller.

4. Granular temperature

The granular temperature is defined as $T(\mathrm{y}) = [\mathrm{T}_x(\mathrm{y}) + \mathrm{T}_y(\mathrm{y})]/2$ for two dimension granular gas system, where $T_i(x) = < (v_i - \overline{v}_i)^2 >$, $(\mathrm{i} = x, y)$. $\overline{v}_i$ is the mean velocity and $\overline{v}_i = 0$ for steady state. This definition is adopted directly from molecular gas, and has been questioned by many people (Evesque, 2002; Baldassarri *et al.*, 2005; Goldhirsch, 2008). Based on the local velocity asymmetric distribution, it is necessary to calculate the temperature components to take the two types of particles into account, for example, $T_i^+ \equiv < v_i^{+^2} >$ (Chen *et al.*, 2012), $(\mathrm{i} = x, y)$.

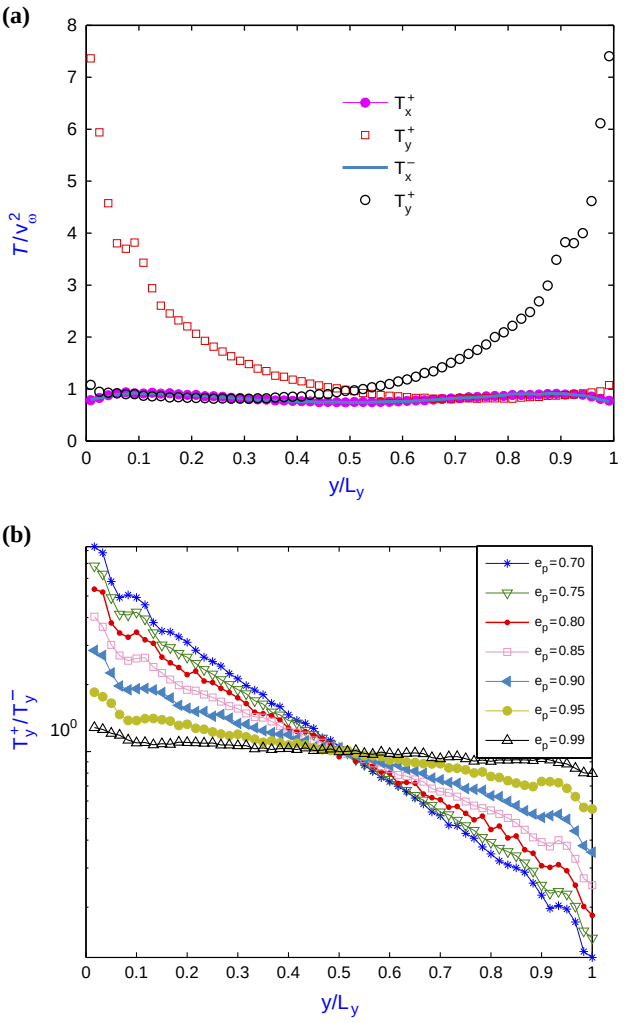
In our previous experiment (Chen *et al.*, 2012), energy equipartition law does not persist for a boundary-driven granular gas. The temperatures $T_y^+(y)$ and $T_{\mathrm{y}}^-(y)$ are not equal to each other along y direction, though $T_x^+(y)$ is equal to $T_x^-(y)$. Figure 5 (a) shows the spatial profiles in our simulation, which agrees with the experiment results, except for the cell center, $T_y^+(y) \neq T_y^-(y)$. This also agrees with our experiment results (Chen *et al.*, 2012). Because of the dissipation in the system, the temperature falls from the boundary to the center.

Furthermore, we observe that with smaller restitution, due to higher dissipation, larger difference between $T_y^+(y)$ and $T_{\mathrm{y}}^-(y)$ appear. However, temperature component in the x direction $T_x^+(y)$ is equal to $T_x^-(y)$ along y-axis. Here we introduce a parameter $\mathrm{h}(y) = T_y^+(y)/T_y^-(y)$, the ratio of the two temperatures $T_y^+(y)$ and $T_{\mathrm{y}}^-(y)$. The spatial profile of $h(y)$ is shown in Figure 5. $h(y) \approx 1$, when $e = 0.999$. With decreasing e, $h(y)$ shows a linear dependence along y-axis in semi-logarithmic coordinates. Assuming $\ln(\mathrm{h}(y)) = -\mathrm{ky}$, we obtain $T_y^+(y)/T_y^-(y) = e^{-ky}$. The ratio of $T_y^+(y)$ and $T_{\mathrm{y}}^-(y)$ drops exponentially. We also plot the slope k of $\ln(h(y))$ in terms of e. It shows that k depends on coefficient of restitution monotonically with a negative slope (Figure 6).

Our results demonstrate that the energy equipartition is broken in a single species granular gas in an event-driven simulation. This result is consistent with our experiment in micro-gravity. Local equilibrium does not exist in a driven granular gas. One temperature could not be able to characterize the system even with zero mean velocity field. The ratio between the two temperature components, T^+ and T^-, is exponential and the coefficient is decreasing with the restitution coefficient.

5. Local heat flux

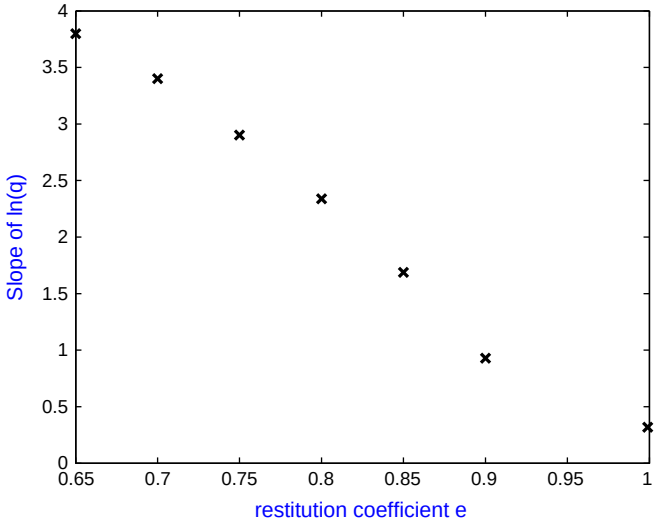
Based on the Fourier law, the heat flux is expressed as $q = -\kappa \nabla T$, where the heat flux is q, κ is the heat conductivity coefficient, T is the temperature. It needs to emphasize that the heat flux here only contains the kinetic part. For an inelastic system (Dufty *et al.*, 1997), the heat flux is predicted by Chapman-Enskog expansions of Boltzmann kinetic equation, $q = -\kappa \nabla T - \mu \nabla n$, where μ is a new transport coefficient for inelastic system, n is the number density. The heat flux, q (Herbst *et al.*, 2005; Dufty *et al.*, 1997), contains



Notes: (a) The four components of the temperature $T_x^+, T_x^-, T_y^+, T_y^-$. The number of particles N is 360, the restitution coefficient is $ep = 0.75$; (b) The semi-logarithmic figure of the T_y^+/T_y^-. The number of particle N is 360. The area fraction $\phi = 0.0126$. The size of the cell is $300 \times 300\, r$, r is the particle radius

Figure 5.
The spatial profiles of the temperature in the simulation

two parts, the free streaming of the particles q_{kin} and the translational energy changes due to collisions q_{int}. Spatial profiles of q_{int} and q_{kin} are nonlinear in the vibrating direction. However, the combination of q_{int} and q_{kin} is found linear. And q_{int} is the same order of magnitude as the kinetic contribution, q_{kin}. This means the collision part is important. In (Herbst *et al.*, 2005), the above generalized Fourier law seems to hold as a constitutive relation for the heat flux. However, for stronger inelasticity (the coefficient $e_p < 0.99$) (Herbst *et al.*, 2005), above theoretical prediction, $q = -\kappa \nabla T - \mu \nabla n$ fails.



Notes: The number of particles N is 360, the area fraction $\phi = 0.0126$. The size of the cell is $300 \times 300\, r$, r is the particle radius

Figure 6.
Slopes k of ln(h(y)) in terms of restitution coefficient

Our simulation here is at low density ($\phi = 0.012$) and high inelasticity $e_p = 0.75$. Moreover, considering two temperatures in the above, the heat flux q_{kin} is divided into four parts: $q_x^+, q_x^-, q_y^+, q_y^-$, for example, $q_x^+(y) = \sum_{i=1}^{N} m\left(v_x^+\right)^2 / 2v_x^+$. Our system is far from equilibrium, but is at a dynamic steady state. Even the temperature looks different from the classical temperature which is discussed in the above section.

An attempt to demonstrate this fact is shown in Figure 7. Indeed from this, one can see that one cannot use a single temperature to describe and calculate the Fourier Law. In Figure 7, spatial profiles of the rescaled heat flux in elastic (Figure 7(a)) and inelastic (Figure 7(b)) situation are compared. Each flux is rescaled by vibrating velocity v_ω to its third power v_ω^3. One sees:

- For near elastic situation, absolute values of $q_x^+, q_x^-, q_y^+, q_y^-$ are equal.
- For inelastic situation, absolute values of q_x^+, q_x^- are equal and symmetric about the center line of the system, but q_y^+ and q_y^- are asymmetric and absolute values of q_y^+ and q_y^- are not equal to each other.
- Keeping the same number of particles and vibration velocity, the values of flux in elastic and inelastic systems are quite different. In addition, this difference is not limited in the boundary layer, but extends to the box center.
- Furthermore, in the case of inelastic system, the number density is not constant and introduce a new heat flux term (the factor ∇n in the equation $q = -\kappa \nabla T - \mu \nabla n$) compared with the Fourier Law. So it is necessary to observe the number density spatial profiles in our simulation. In previous works (Chen *et al.*, 2012), n_x^+ and n_x^- are equal to each other, while the two components of the number density , n_y^+ and n_y^- are not equal to each other except at the box center point (y-axis is the vibration direction). The reason can be explain as follows, near the left boundary of the cell, the mean velocity $< v_y^+ >$ (particles moving away from

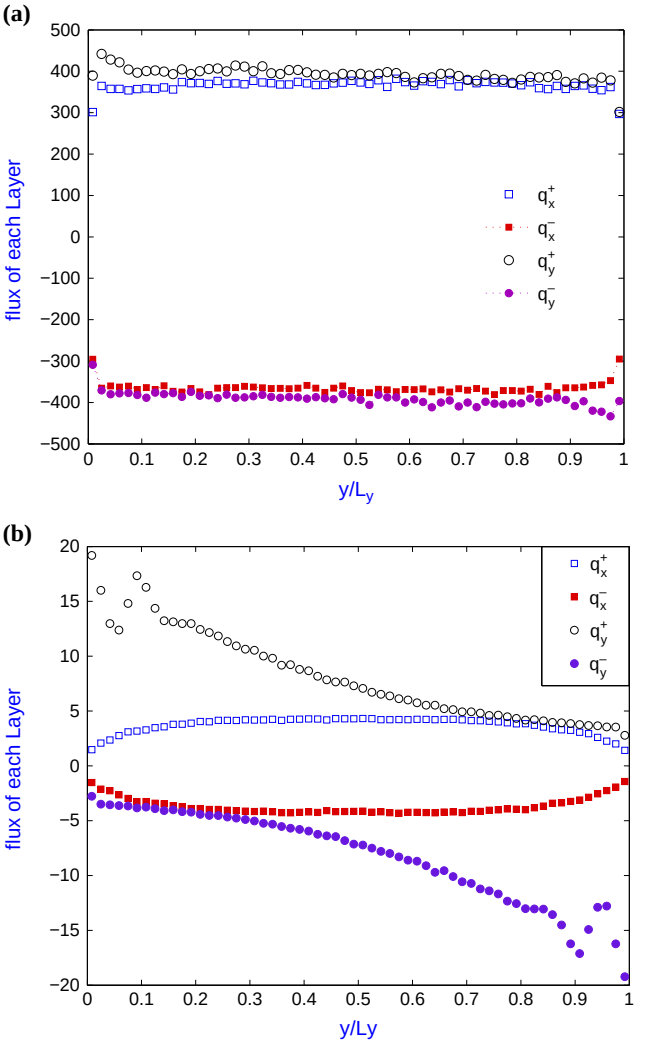
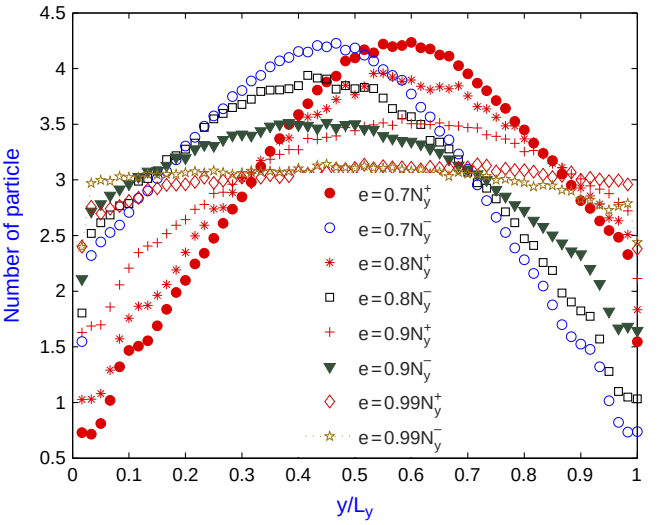


Figure 7.
The comparison of spatial profiles of the four components of the rescaled heat flux (q_x^+, q_x^-, q_y^+, q_y^-) q_{kin} with $e_p = 0.99$ (left) and $e_p = 0.75$ (right)

Notes: The number N of particles is $N = 360$, the area fraction $\phi = 0.012$. The size of the cell is $300 \times 300\, r$, r is the particle radius

the wall, which gain velocity from the wall) is larger than that of $< v_y^- >$ (particles moving toward the wall, which lose energy through particle-particle collisions). Since there is no velocity field in a steady state, the mean velocity ($< v_y > = n^+ < v_y^+ > + n^- < v_y^- >$) shall be zero, where n^+ is the particle number with $v_y{}^+$ and n^- is the particle number with velocity v_y^-. So we will have $n^+ < n^-$. Our simulation results confirm the spatial profiles of n_y^+ differ from n_y^- in various coefficients of restitution (shown in Figure 8).

- Furthermore, the more dissipation, the sharper density peak. The larger the difference between n_y^+ and n_y^-, the larger the heat flux exchanges. This is



Notes: The number of particle N is 360, the area fraction $\phi = 0.012$. The size of the cell is $300 \times 300\,r$, r is the particle radius

Figure 8.
The spatial profiles of the two components of the numbers of particles n_y^+ and n_y^- in various coefficients of restitution

consistent with the difference between the temperature T_y^+ and T_y^-. All these results demonstrate the necessity to analyze the heat flux in details and especially the contribution of both the positive and the negative parts. This point should be considered in the inelastic heat flux theory and need to be explored in the future work. This result highlights difference of heating function between the two boundary walls.

6. Concluding remarks

In this paper, we have discussed the asymmetry of the local velocity distribution in the vibrating direction. We have measured the skewness of the local velocity distribution in terms of various restitution coefficients ($0.999 < e_p < 0.65$). For the elastic system, the skewness is nearly uniform so that $S = 0$ everywhere. For inelastic system, except at the cell center, the skewness is non-zero. In left side of the box, the skewness is positive and decreases with the distance away from the boundary. In the right side of the box, the skewness is negative, and increases away from the boundary. The spatial profiles of the skewness indicate that this asymmetry is a bulk effect. Furthermore, we have shown that the value of skewness increases when the restitution coefficient is decreased; this demonstrates this long range boundary effect is related to the system dissipation.

By using a simplified phenomenological model (superposition of two Gaussian functions), an expression of skewness can be obtained, which is consistent with this long range boundary effect, consistent with our simulation data and consistent with our previous Airbus experiment results. This confirms that boundary heating mechanism and the dissipation property of the system lead to this long range boundary effect. The temperature evolution has been also studied using the simulation. Study needs to define two different temperature range and variations in x and y.

Furthermore, one shall split the temperature problem in the y direction (vibration direction) to introduce two different temperatures T_y^+ and T_y^-. We have calculated

the two temperature from the two components: the positive and negative temperature. We have found the energy equipartition does no longer persist in our simulation. Moreover, the ratio of two temperatures drops exponentially.

Finally, we have discussed the heat flux in our view and find the difference between the classical system and granular system. These imply that a new point of view is needed for exploring the properties of a granular system.

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Corresponding author
Professor Meiying Hou can be contacted at: mayhou@iphy.ac.cn