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Frictional effect of bottom wall on granular flow through an aperture on a conveyor belt

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ABSTRACT

Horizontal flow of discrete objects through a bottleneck is prevalent and important in nature and industry. In this paper, we experimentally investigate the frictional effect of the bottom wall on the flow rate of granular particles passing through an aperture on a conveyor belt, since the bottom wall is always assumed to be trivial and the issue remains elusive. The flow rate monotonically increases with the coefficient of friction, finally approaching a saturation plateau. The bottom wall has an obvious effect on the flow rate. We show that the lateral motion of particles on both sides of the aperture, which is influenced by the coefficient of friction and can be converted into an effective aperture width, is responsible for the variation of the longitudinal flow rate. Our findings may provide a new way to manipulate the horizontal flow rate, particularly for the case where the aperture has to be fixed.

1. Introduction

The horizontal flow of discrete objects through a narrow aperture, namely through a bottleneck, is ubiquitous in a broad range of fields, such as emergency evacuation of panicking crowds [1–3], crowd behavior of escaping animals through a narrow exit [4–6], clogging of micro- or nanoparticles at an aperture in a microfluidic system [7], and traffic jam in a narrow entrance or exit [8]. For the crowd behavior and motion of particles through an aperture, there are two main research subjects which have drawn considerable interest. One topic focuses on the transition between the flowing and clogging regimes [9–12]. These studies reveal a unified power-law tail in the probability density function of time lapses, measured between consecutive individuals through a bottleneck, and potentially shed light on crowd stampedes induced by panic.

The investigation of the dependence of a horizontal flow rate through an aperture on systematic variables is another theme which attracts attention. Unlike fluid flows, which are controlled by pressure, experiments on flows of discrete material confirm that the flow rate driven by a horizontal conveyor belt is independent of the pressure at the region of the aperture [13,14]. The flow rate presents two distinct regimes depending on the belt velocity. At a low belt velocity the flow rate is linearly proportional to the belt velocity and to the aperture width. However, beyond a critical velocity, the flow rate becomes independent of the belt velocity and is proportional to an effective aperture width in correspondence with a two-dimensional Beverloo scaling [15,16]. Also, the effect of the initial packing fraction of particles on the flow rate has been also investigated. For initially loose systems, the time-dependent flow rate is directly altered by the local packing fraction of granular materials [17].

The aperture size, the driven velocity and the initial packing fraction of particles have been considered as key factors for the horizontal flow rate, but the influence of bottom walls is generally neglected except that very few works studied it indirectly [18]. The experimental measurements are interpreted as if the bottom wall, often with a smooth surface, does not play a major role on the flowing dynamics. However, this assumption is far from being straightforward and in our opinion warrants more investigation.

In this work, we experimentally study the frictional effect of the bottom wall on the flow rate of granular particles driven through an aperture on a conveyor belt. Specifically, we measure the flow rate for bottom walls with different coefficients of friction. We define and

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measure a blocking width which quantitatively characterizes the lateral motion of particles on both sides of the aperture. Using the blocking width, we redefine an effective aperture width that is directly related to the flow rate, and thus discover the underlying mechanism for the dependence of the flow rate on the coefficient of friction of the bottom wall. In addition, we analyze the relation between the flow rate and the coefficients of friction with an empirical law. The explanations from the two perspectives are consistent.

2. Experimental setup

The experimental apparatus consists of a conveyor belt (length 160 cm, width 50 cm) above which a confining acrylic frame (length 90 cm, width 44 cm) is maintained at a fixed position. A motor drives the belt at a constant velocity $V = 15.1$ cm/s. 1800 Plexiglas disks are put in the two-dimensional acrylic frame, as shown in Fig. 1(a). The geometry of the disk is measured as follows: thickness $t = 5.00 \pm 0.08mm$, diameter $D = 10.92 \pm 0.08mm$, and mass $m = 0.389 \pm 0.006g$. A rectangular aperture with a width A is located at the center of the bottom wall of the frame. Granular flow is maintained continuously since the aperture is larger than the critical size where clogging occurs ($A > 5D$).

The coefficient of friction of the bottom wall is modified by pasting sandpapers with various roughnesses on the bottom wall. Note that the bottom wall is neither the surface of the conveyor belt nor the side wall of the confining frame. The bottom wall is the flat bottom of the acrylic frame at which the aperture is located (Fig. 1 (b)). In this experiment, we use six types of sandpaper to change the coefficient of friction, which is quantitatively determined as follows: we paste the sandpaper on the surface of a flat glass plate, and glue several disks as feet on a sleigh. We put the sleigh on the plate surface and the sleigh feet keep contact with the surface. We slowly raise the plate until the sleigh starts to slide along the plate surface covered by the sandpaper. We measure the inclined angle α and obtain the coefficient of friction $\mu = \tan \alpha$ of the sandpaper with respect to the disks.

Before the flow starts, the initial packing state of the disks is prepared in accordance with the procedure outlined below. First, the disks are deposited in a random packing status, namely in a disorder manner, inside the confining frame. Then the belt is moved at a low velocity until all disks are packed against the bottom wall on which the aperture is kept closed ($A = 0$). Finally, the packing fraction of this initial configuration reaches a stable value $\phi_0 = 0.84 \pm 0.02$.

After the preparation is completed, the aperture is opened to the desired width A and the belt is moved at a selected constant velocity V. A high-resolution digital camera hung above the frame records the temporal evolution of the disk number inside the frame. After photos are taken, an intensity threshold converts the pictures into binary images. The number of disks $N_{in}(t)$ inside the confining frame at time t is calculated from white pixels in the images. We then obtain the number of disks flowing out of the aperture at time t, $N(t) = N_0 - N_{in}(t)$, where N_0 is the total number of the disks. The relation between $N(t)$ and t shows a linear form (Fig. 1 (c)). Through linear fitting to the $N(t)$ curve, the flow rate Q, the slope of the fitting line, can be extracted. It is worthwhile to note that the first three seconds after the first particle discharging from the aperture is removed from the $N(t)$ curve for fitting. In this way we can ensure that a steady flow regime has been reached for the measurement of the flow rate. The measurement stated above is repeated six times for each data point, and the flow rate is computed as an arithmetic average of these trials.

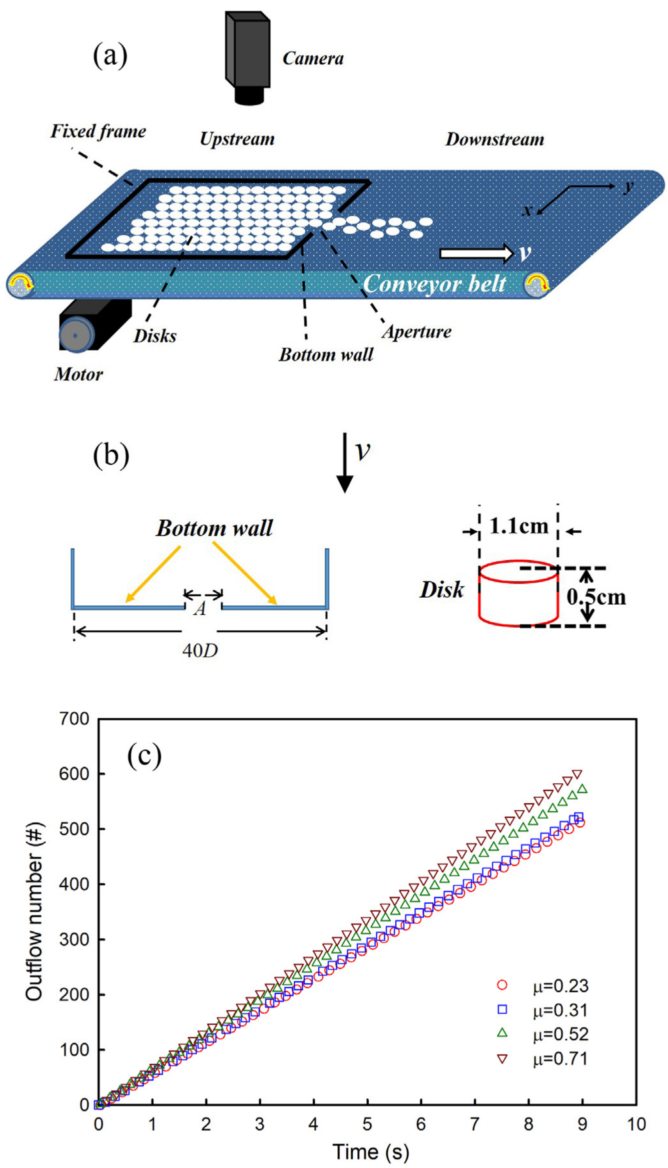

Fig. 1. (a) Schematic diagram of the experimental setup. A conveyor belt moves at a constant velocity V (indicated by a white arrow), and disks confined in a stationary frame flow through an aperture centered at the bottom wall. (b) Illustration of the bottom wall. By using different sandpaper on the bottom wall, the coefficient of friction μ of the bottom wall is controlled. (c) Representative time dependence of the number of outflowing disks is measured for different μ, showing a nearly linear form. The flow rate Q is extracted through linear fitting. The aperture width $A = 6D = 6.6$ cm, and the conveyor belt velocity $V = 15.1$ cm/s.

3. Results

The flow rate is affected by the coefficient of friction of the bottom wall. Here, the flow rate Q shows a monotonic increasing tendency with the coefficient of friction μ, as shown in Fig. 2. With continuously increasing μ, Q gradually approaches a saturation plateau Q_{sat} for the roughest bottom, which is about 18% larger than the flow rate (Q_0) for the smoothest bottom we can find now ($\mu = 0.23$), see the inset in Fig. 2.

We experimentally observe that disks on a smooth bottom wall easily move laterally and the disks on both sides of the aperture often squeeze above the aperture, as shown in Fig. 3 (a). In contrast, the disks on a rough bottom wall remains almost stationary and lateral motion rarely occurs. This suggests that the coefficient of friction of the bottom wall can directly influence the lateral motion of the disks in contact with the bottom wall. We speculate that the protrusion of the disks due to the lateral motion practically acts as a temporary obstacle which can reduce instantaneous flow rate significantly. It is experimentally proven (Fig. 3 (b)) that the instantaneous flow rate at a smooth bottom wall ($\mu = 0.23$) declines more abruptly and frequently than that at a rough bottom wall ($\mu = 0.91$). The correlation between the instantaneous flow

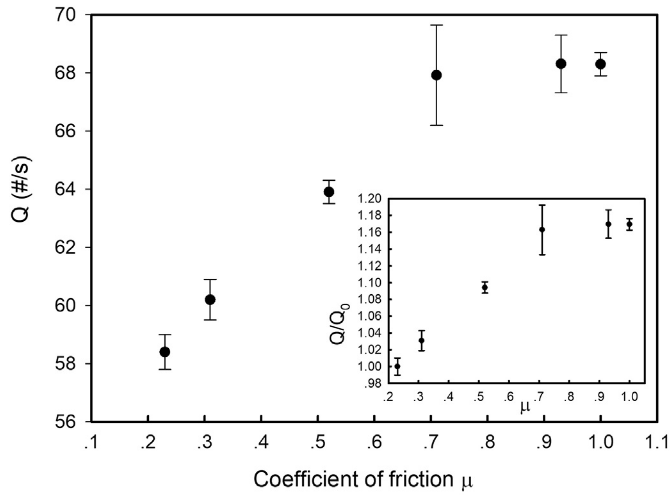

Fig. 2. Flow rate Q as a function of the coefficient of friction of the bottom wall μ. Every point is averaged from six measurements. Inset: normalized flow rate is plotted as a function of the coefficient of friction. Q_0 corresponds to the lowest flow rate in the main panel. The aperture width $A = 6D = 6.6$ cm, and the conveyor belt velocity $V = 15.1$ cm/s.

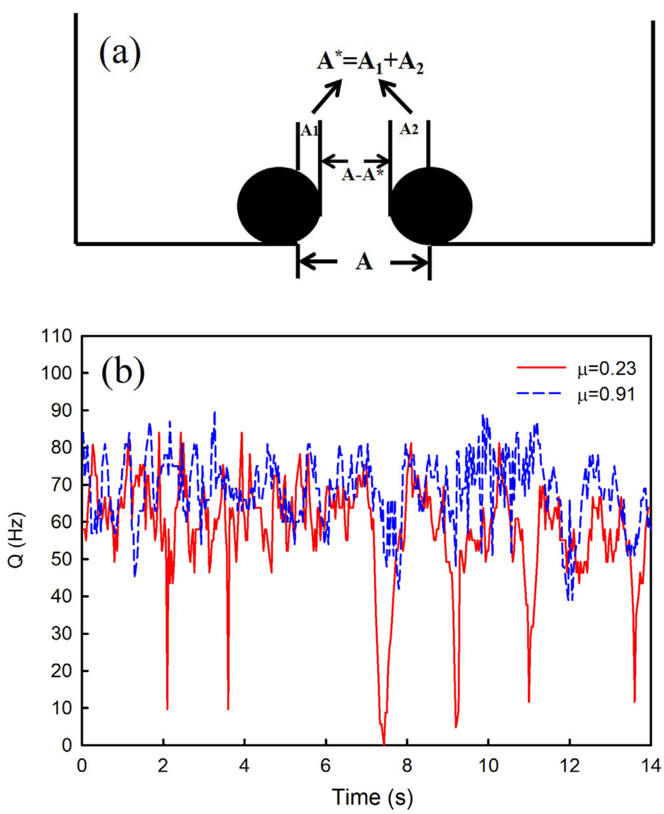

Fig. 3. (a) Schematic diagram of the disks on both sides of an aperture squeezing into the aperture. The sum of the protruding part is defined as a blocking width A^*. (b) Time evolutions of the instantaneous flow rate are measured at $\mu = 0.23$ and $\mu = 0.91$. The instantaneous flow rate has obvious, large, and frequent fluctuations at the case of $\mu = 0.23$, which implies that the lateral protrusion of the disks can effectively block the flow. The aperture width $A = 6D = 6.6$ cm, and the conveyor belt velocity $V = 15.1$ cm/s.

rate Q and instantaneous blocking width A^* is also measured and supports the speculation, see the supplementary materials. The fact that the protrusion is related to the flow rate makes it necessary to quantitatively characterize the size of the protrusion.

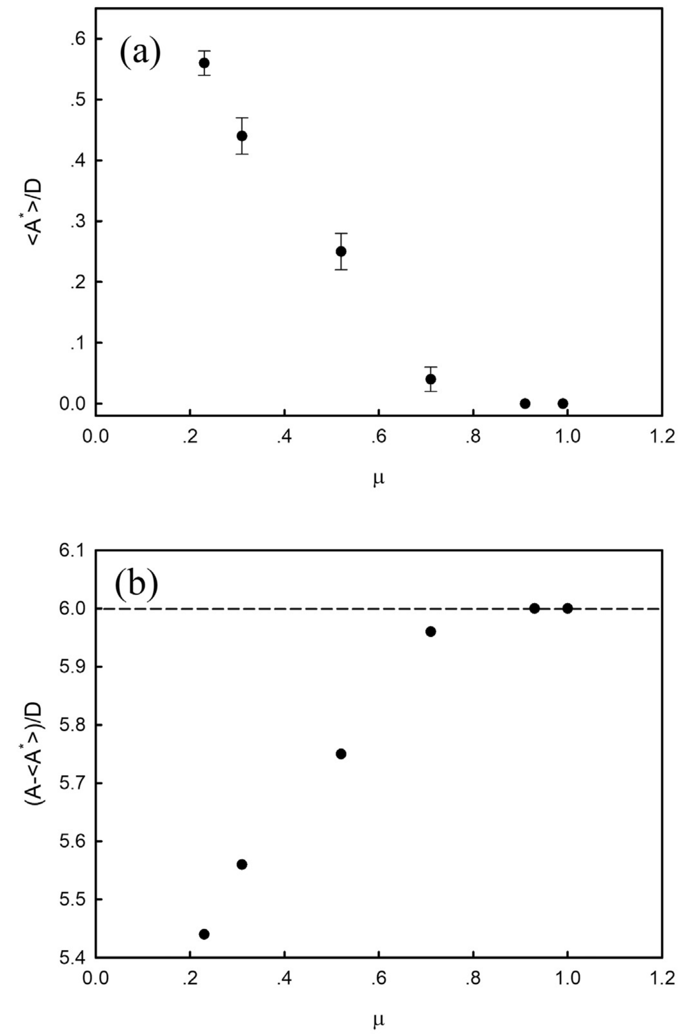

Fig. 4. (a) Normalized average blocking width $<A^*>/D$ as a function of the coefficient of friction, D is the diameter of one disk. (b) Normalized effective aperture width $(A - <A^*>)/D$ as a function of the coefficient of friction. Dashed line indicates the upper bound of the effective aperture width. The aperture width $A = 6D = 6.6$ cm, and the conveyor belt velocity $V = 15.1$ cm/s.

A series of snapshots recording the motion of disks near an aperture during a steady flow were taken by a digital camera. For each photograph, total protrusion of the disks on both sides of the aperture, A^*, called an instantaneous blocking width, can be extracted by an imaging processing technique. The mean of A^* in the steady flow, defined by $<A^*> = \frac{1}{N}\sum_{i=1}^{N} A_i^*$, A_i^* the blocking width in ith snapshot, is used to quantify the size of the protrusion. Each average $<A^*>$ is averaged on 300 instantaneous blocking width A^*. Because the average flow rate in each measurement is not only influenced by the abrupt change of the instantaneous A^*, but also determined by all instantaneous A^* during this measurement. Thus, we choose a video recording steady flow of 5 s in each measurement, pick a photograph every 0.1 s in the 5 s video and extract the instantaneous blocking width A^* from each photograph. The measurement for average flow rate repeats 6 times, and we totally used 300 photographs to obtain the average blocking width $<A^*>$.

The relationship between the mean blocking width $<A^*>$ and the coefficient of friction μ is shown in Fig. 4(a). The mean blocking width monotonically decreases with increasing coefficient of friction. As the coefficient of friction is higher than a certain threshold, $<A^*>$ approaches zero, and the lateral protrusion disappears. Interestingly, the friction threshold $\mu = 0.71$ in Fig. 4(a) is the same as that in Fig. 2, where the flow rate asymptotically tends to a constant beyond this particular friction threshold. This implies that there is probably a relation between the flow rate and mean blocking width, or the lateral motion of disks on the bottom wall.

The physical meaning of $A - \langle A^* \rangle$ is more straightforward, which can be considered as an effective width of an aperture for granular flow. The coefficient of friction dependence of the effective aperture width is plotted in Fig. 4(b). The effective aperture width increases with the coefficient of friction and eventually reaches a saturation plateau at 6D, i.e. the aperture width. As we know, the effective aperture width is positively related to the flow rate. For the smoothest bottom wall, the effective aperture width is minimum, around 5.4D, naturally corresponding to the lowest flow rate.

At this point, the saturation plateau in the flow rate can be explained by the analysis above. As the coefficient of friction is increased, the disks on the bottom wall become more and more difficult to move laterally, the lateral motion of disks near the aperture diminishes, and the effective aperture width will gradually increase accordingly. Thus, the flow rate will rise with the coefficient of friction. As the coefficient of friction goes up to a threshold, the disks on the bottom wall remain motionless and the lateral motion ceases regardless of the coefficient of friction. The effective aperture width reaches its upper limit and stays constant. Consequently, the flow rate stays the same and the saturation plateau is obtained. We use an experiment to validate this argument. On the bottom wall is glued a double-sided adhesive tape, which is so sticky that the coefficient of friction can be seen as practically infinite. The disks in the lowest layer are completely glued on the bottom wall and lateral motion does not occur. The flow rate in this case is measured to be $Q = 68.4 \pm 0.4\ \mathrm{s}^{-1}$, which is almost the same as the flow rate at $\mu = 0.91$ ($Q = 68.3 \pm 0.9\ \mathrm{s}^{-1}$) and $\mu = 0.99$ ($Q = 68.4 \pm 0.3\ \mathrm{s}^{-1}$), instead of continuously growing indefinitely. To summarize, the coefficient of friction on the bottom wall alters the effective aperture width, and finally results in variation of the flow rate.

4. Discussions

In previous work, an empirical law was presented to describe the flow rate through an aperture on a conveyor belt [13,17].

$$Q = \frac{4\phi V}{\pi D^2}(A - kD) \tag{1}$$

Where ϕ the system packing fraction, V the conveyor belt velocity, D the diameter of each disk, A the width of the aperture, and k is a correction factor. Eq. (1) is equivalent to the 2D Beverloo's law in which the typical velocity $\sqrt{gA_{eff}}$, understood as the typical velocity of the particles at the aperture, is replaced by the belt velocity V. It is interesting to note that this empirical law is even valid for a system with a small aperture where clogging and intermittent flow occur [10].

Our experimental system is a continuous flow system with packing fraction $\phi = 0.84 \pm 0.02$, to which the empirical law is applicable. In Fig. 2, all external parameters in the empirical law, such as the system packing fraction and the belt velocity, are fixed, except for the

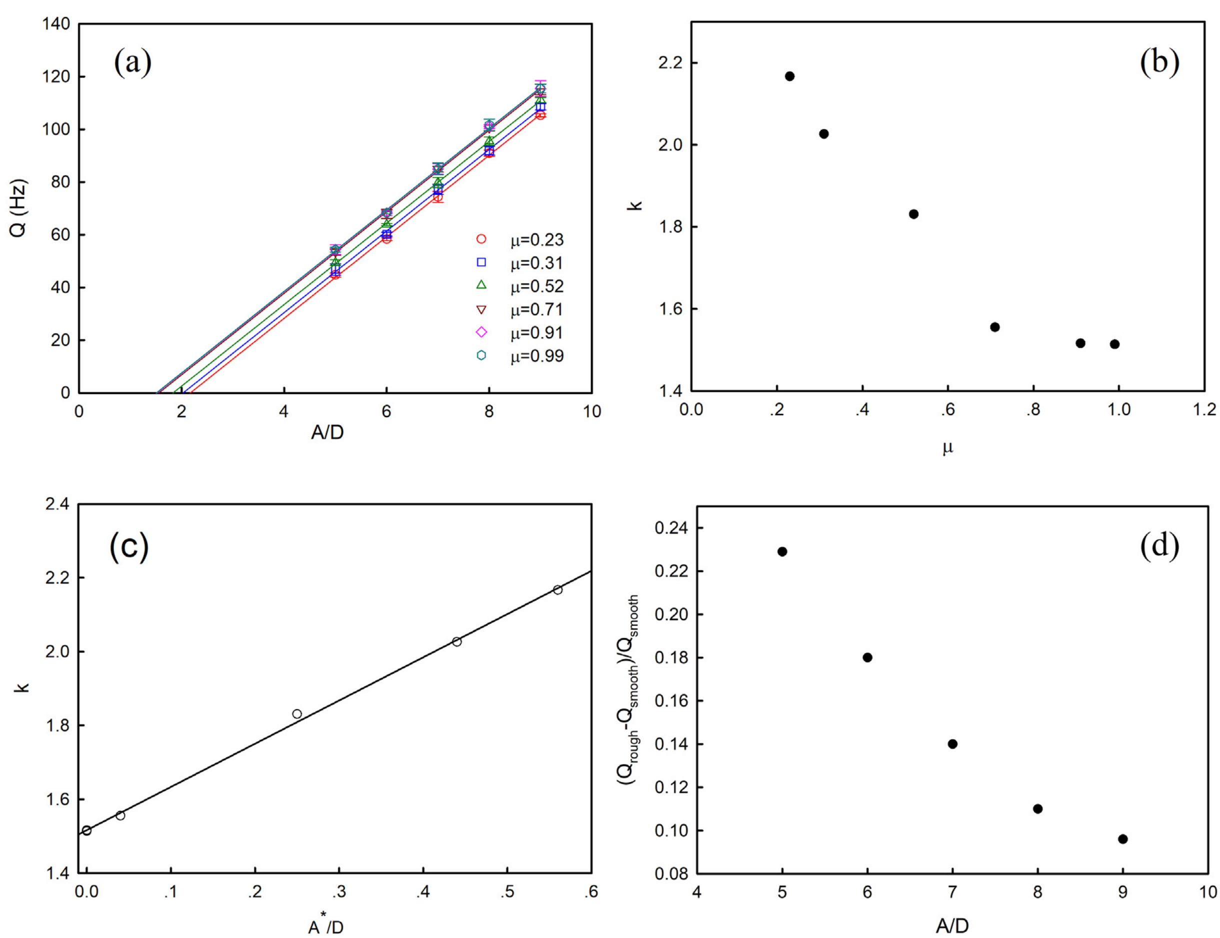

Fig. 5. (a) Flow rate as a function of a normalized aperture width (scaled by the diameter of the disk) at different coefficients of friction. Straight lines are fitting lines. (b) k as a function of the coefficient of friction μ. Beyond a critical value of the coefficient of friction, k reaches a stable value. (c) k vs A^*/D, showing a nearly linear form. (d) Variation ration of the flow rate between the roughest bottom and the smoothest bottom is plotted as a function of the aperture width in a unit of particle diameter.

coefficient of friction of the bottom wall. According to the empirical law, we consider that the coefficient of friction of the bottom wall influences the correction factor k, namely $k = k(\mu)$, and thus causes the change in the flow rate.

The experiment shown in Fig. 5(a) and (b) validate this approach. With a given $V = 15.1$ cm/s, the aperture width dependence of the flow rate is measured at different coefficients of friction, showing a linear form $Q \propto \frac{A}{D}$, which agrees well with the empirical law here rewritten as $Q = \frac{4\phi V}{\pi D}\left(\frac{A}{D}\right) - \frac{4\phi V}{\pi D}k$. We use linear fitting in Fig. 5(a) to find the intercept of the fitting straight line on x axis, and then determine k. k monotonically decreases with the increasing coefficient of friction μ, and remains nearly constant when μ is larger than a certain value, namely around $\mu = 0.71$, see Fig. 5 (b).

The coefficient of friction of the bottom wall apparently influences the correction factor k in the empirical law. Interestingly, k will not vanish, even if the coefficient of friction is very large. We even use double-sided adhesive tape, here assuming an infinite coefficient of friction, to glue on the bottom wall, and k still exists and is the same as the plateau value in Fig. 5(b). In previous studies, k was considered to be mainly originated from the confinement effect of finite size of the aperture [13,18]. However, the coefficient of friction μ of the bottom wall also contributes to k, and the contribution is obvious and not negligible, as suggested by Fig. 5(b). Accordingly, when the bottom wall is smooth (small μ), part of k comes from the confinement effect of the aperture, and part of k is attributable to the coefficient of friction μ. In contrast, k mainly comes from the confinement effect of the aperture for the case of a bottom wall with a large μ, where k maintains nearly constant regardless of μ.

With using Figs. 4(a) and 5(b), the relationship between the correction factor k and mean blocking width is obtained, see Fig. 5(c). There is a clearly linear positive correlation between both quantities, implying that the interpretations from the two perspectives are consistent.

Finally, it should be pointed out that the variation ratio of the flow rate between the roughest bottom and the smoothest bottom depends on the aperture width A, as shown in Fig. 5(d). The ratio monotonically reduces with the aperture width. The result can also be derived from the empirical Eq.1 $Q = \frac{4\phi V}{\pi D^2}(A - kD)$. Substitute the empirical equation into the variation ratio, then $\frac{Q_{rough} - Q_{smooth}}{Q_{smooth}} = \frac{k(\mu_{smooth}) - k(\mu_{rough})}{\frac{A}{D} - k(\mu_{smooth})}$, $k(\mu_{smooth})$ and $k(\mu_{rough})$ are constants and given in Fig. 5 (b). Therefore, when the aperture width A rises, the variation ratio decreases, $\frac{Q_{rough} - Q_{smooth}}{Q_{smooth}} \propto \frac{1}{A/D}$.

5. Conclusion

In summary, we experimentally investigate the frictional effect of the bottom wall on the flow rate of granular particles through an aperture on a conveyor belt. The flow rate increases with the coefficient of friction of the bottom wall, and eventually reaches a saturation plateau. The explanation for the apparent effect is that the friction on the bottom wall converts the lateral motion of disks on both sides of the aperture into a variation of the longitudinal flow rate by changing the effective aperture width. Analysis of photographic recording shows that the lateral motion of disks on both sides of the aperture is indeed influenced by the coefficient of friction on the bottom wall. A parameter, called a blocking width, is presented to quantify the extent of the lateral motion. By using the blocking width, we define an effective aperture width which is positively related to the flow rate.

Furthermore, we explain the experimental results from another prospective. We analyze the results on the basis of an empirical formula. It is found that the coefficient of friction on the bottom wall influences a correction factor k in this empirical equation, thereby affecting the flow rate. There is a linear positive correlation between the mean blocking width $\langle A^* \rangle$ and the correction factor k, implying that the interpretations from the two aspects are consistent.

Tuning the coefficient of friction is equivalent to manipulation of the aperture width. Therefore, manipulation of friction on the bottom wall provides an alternative method to control the flow rate, particularly for the case where the aperture has to be fixed. Even more interestingly, tuning bottom wall friction is very likely to induce a transition between a continuous flowing to an intermittent flowing regime, and even a jamming state for an appropriate aperture size. Finally, the reported frictional effect can be seen as a strong test for any theoretical model or numerical simulation describing the dense shear flow of granular assemblies on a horizontal conveyor belt.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.powtec.2020.03.067.

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